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Left orderability, foliations, and transverse (π_1, \mathbb{R}) structures for 3-manifolds with sphere boundary

Let M be a closed orientable irreducible 3-manifold such that $\pi_1(M)$ is left orderable.

(a) Let $M_0 = M - \text{Int}(B^3)$, where B^3 is a 3-ball in M . We show that there exists a Reebless co-orientable foliation \mathcal{F} in M_0 , whose leaves may be transverse to ∂M_0 or tangent to ∂M_0 at their intersections with ∂M_0 , such that \mathcal{F} has a transverse $(\pi_1(M_0), \mathbb{R})$ structure and that \mathcal{F} is analogous to taut foliations (in closed 3-manifolds) in the following sense: there exists a compact 1-manifold (i.e. a finite union of properly embedded arcs and/or simple closed curves) transverse to \mathcal{F} that intersects every leaf of \mathcal{F} .

(b) We have a process to produce a foliation \mathcal{F} as given in (a), which depends on the choice of a left-invariant order of $\pi_1(M)$ and certain fundamental domain Γ of M . If M admits a taut foliation that has a transverse $(\pi_1(M), \mathbb{R})$ structure, then some resulting foliation of our process can extend to a taut foliation in M that has a transverse $(\pi_1(M), \mathbb{R})$ structure. If M admits an \mathbb{R} -covered foliation, then some resulting foliation of our process can extend to an \mathbb{R} -covered foliation in M . Furthermore, we conjecture that every resulting foliation of our process can extend to a taut foliation in M that has a transverse $(\pi_1(M), \mathbb{R})$ structure.