Geometric Tomography and Microlocal Analysis
Tomographie géométrique et analyse microlocale
(Org: **Suresh Eswarathasan** (Dalhousiie) and/et **Alina Stancu** (Concordia))

MARIA ALFONSECA, North Dakota State University
Solutions to the 5th and 8th Busemann-Petty problems near the ball

In this talk we apply classical harmonic analysis tools, such as singular integrals and maximal functions, to two Busemann-Petty problems, numbers 5 and 8 in their famous list of 10 problems about convex bodies. We obtain local affirmative results for convex bodies that are close to the Euclidean ball in the Banach-Mazur distance.

GAIK AMBARTSOUMIAN, University of Texas at Arlington
Inversion and Symmetries of the Star Transform

The star transform is a generalized Radon transform mapping a function of two variables to its integrals along "star-shaped" trajectories, which consist of a finite number of rays emanating from a common vertex. Such operators appear in mathematical models of various imaging modalities based on scattering of elementary particles. The talk presents a comprehensive study of the inversion of the star transform. We describe the necessary and sufficient conditions for invertibility of the star transform, introduce a new inversion formula and discuss its stability properties.

This is a joint work with Mohammad Javad Latifi Jebelli (University of Arizona).

RALUCA FELEA, Rochester Institute of Technology
Microlocal analysis of the dense array borehole seismic data

In the seismic scattering problems, acoustic waves are generated at the surface, they scatter off the features in the subsurface, and return to the surface to be detected by receivers. In the dense array borehole seismic inverse problems, the sources are located on the surface and the receivers are located in a borehole. We will describe the microlocal properties of the forward operator F, which maps the image to the data, in the presence of fold caustics. We will show that F is a Fourier Integral Operator with singularities like submersions with folds and cross caps. To find the image, one studies the normal operator F^*F, and in this case, F^*F is a paired Lagrangian operator $(I^{p,l}(\Delta, C))$ which will produce artifacts. We will describe these artifacts and find their strength.

ALLAN GREENLEAF, University of Rochester
Sobolev estimates for multilinear Radon transforms via partition optimization

Multilinear generalized Radon transforms and oscillatory integral operators occur in many settings. I will describe a method for obtaining Sobolev space bounds for such multilinear operators by optimizing over a collection of linear estimates. This approach was motivated by problems in geometric measure theory/combinatorics, and I will give examples of these applications, but this technique might be applicable for other problems involving multilinear operators, such as in scattering theory and inverse problems. This is joint work with Alex Iosevich and Krystal Taylor.

ALEX KOLDOBSKY, University of Missouri-Columbia
Inequalities for the Radon transform on convex sets

We look at some problems of convex geometry from a more general point of view, replacing volume by an arbitrary measure. This approach leads to new general properties of the Radon transform on convex bodies including extensions of the Busemann-Petty problem and slicing inequalities.

VENKATESHWARAN KRISHNAN, TIFR Centre for Applicable Mathematics
Higher order Reshetnyak formulas for the ray transform of symmetric tensor fields in Sobolev spaces

The ray transform of a symmetric m-tensor field is defined as integrals of the tensor field along lines in Euclidean space. In this talk, we derive higher order Reshetnyak formulas for the ray transform of symmetric fields in Sobolev spaces. These are Plancherel-type isometry relations involving certain weighted Sobolev norms of a tensor field and that of its ray transform. This is joint work with Vladimir Sharafutdinov.

FRANÇOIS MONARD, University of California Santa Cruz
Sampling issues for the X-ray transform on simple surfaces

Abstract: On a Riemannian manifold-with-boundary, the geodesic X-ray transform maps a function to the collection of its integrals over all geodesics through the domain, with applications to medical imaging and seismology. In the literature, it is now well-known that injectivity, stability (or mild instability) and inversion formulas hold at the continuous level, for example when (M, g) is a 'simple' surface. By 'simple' we mean (i) no infinite-length geodesic, (ii) no conjugate points, (iii) strictly geodesically convex boundary, arguably the most inversion-friendly case.

In this talk, I will discuss the issue of proper discretizing and sampling of the X-ray transform, addressing the following: (a) Given a bandlimited function f, what are the minimal sampling rates needed on its X-ray transform $I_0 f$ for a faithful (=unaliased) recovery? (b) In the case where data is sampled below the expected requirements, can one predict the location, orientation and frequency of the artifacts generated?

The main tools to answer (a)-(b) are a combination of a reinterpretation of the classical Shannon-Nyquist theorem in semi-classical terms, as initiated in [P. Stefanov, https://doi.org/10.1137/19M123868X], and an accurate description of the canonical relation of the X-ray transform viewed as a (classical, then semi-classical) Fourier Integral Operator. The answer also depends on geometric parameters of the surface (e.g., curvature and boundary curvature), and on the coordinate system used to represent the space of geodesics. Several (unaliased and aliased) examples will be given throughout.

Joint with Plamen Stefanov (Purdue). Preprint available at https://arxiv.org/pdf/2110.05761.pdf

SERGII MYROSHNYCHENKO, Lakehead University
On the recognition of polyhedra

We discuss two dual approaches for the recognition of convex polyhedra in terms of non-central sections and point projections. The results generalize V. Klee's theorems that provide sufficient (and necessary) conditions for a convex body to be a polytope.

TODD QUINTO, Tufts University
Microlocal analysis of Fourier Integral Operators in scattering tomography

We present a novel microlocal analysis of generalized Radon transforms (which are Fourier integral operators) that describe the integrals of L^2 functions of compact support over surfaces of revolution in $\mathbb{R}^n$ of C^∞ curves defined by a smooth function $q : [0, \infty) \to [0, \infty)$. We show that the Radon transforms are elliptic Fourier Integral Operators (FIO) and provide an analysis of the left projections Π_L. Our main theorem shows that Π_L satisfies the semi-global Bolker condition if and only if $g = q'/q$ is an immersion. An analysis of the visible singularities is presented, after which we derive Sobolev smoothness estimates for these generalized Radon FIO.

Our theory has specific applications in Emission Compton Scattering Tomography (ECST) and Bragg Scattering Tomography (BST). We show that the ECST and BST integration curves (or surfaces in $\mathbb{R}^3$) satisfy the semi-global Bolker Condition and provide simulated reconstructions from ECST and BST data. Additionally we give example “sinusoidal" integration curves which do not satisfy Bolker and provide simulations of the image artifacts. The observed artifacts in reconstruction are shown to align exactly with our predictions.

This is joint work with James Webber (Brigham and Women's Hospital, formerly Tufts University).

DMITRY RYABOGIN,
On an equichordal property of a pair of convex bodies

Let $d \geq 2$ and let K and L be two convex bodies in $\mathbb{R}^d$ such that $L \subset \operatorname{int} K$ and the boundary of L does not contain a segment. If K and L satisfy the $(d+1)$-equichordal property, i.e., for any line l supporting the boundary of L and the points $\{\zeta_\pm\}$ of the intersection of the boundary of K with l,

$$\operatorname{dist}^{d+1}(L \cap l, \zeta_+) + \operatorname{dist}^{d+1}(L \cap l, \zeta_-) = 2\sigma^{d+1}$$

holds, where the constant σ is independent of l, does it follow that K and L are concentric Euclidean balls? We prove that if K and L have C^2-smooth boundaries and L is a body of revolution, then K and L are concentric Euclidean balls.

KATERYNA TATARKO, University of Waterloo
L_p Steiner formula and its coefficients

In this talk, we explore L_p Steiner formula for the L_p affine surface area. We introduce the coefficients that arise in this formula that we call L_p-Steiner quermassintegrals and discuss their properties. In analogy with classical quermassintegrals, it turns out that they also possess some nice properties. In particular, they are new valuations on the set of convex bodies which seem not to have been observed before. This is joint work with E. Werner.

ANDRAS VASY, Stanford University
The inverse problem for the geodesic X-ray transform

In this talk I will give an overview of some recent developments in the study of the geodesic X-ray transform starting with the spatially localized approach introduced by Uhlmann and the speaker. In particular, I will give an overview of this method, extensions, applications as well as discussing a new twist on it.

YIRAN WANG, Emory University
Microlocal analysis in cosmological X-ray tomography

Cosmic Microwave Background (CMB) is the radiation remnant from the Big Bang and is considered to be a primary source of information regarding the early universe. From the work of Sachs and Wolfe (1967), it is known that the linearization of the CMB redshift leads to an X-ray transform of the gravitational perturbations, called the light ray transform. We discuss recent results about the transform and the recovery of metric perturbations of scalar type. In particular, we focus on the microlocal properties of the light ray transform and its connection to hyperbolic type PDEs. Furthermore, we consider the transport theory of the light ray transform and study inverse source problems for the Boltzmann equation in the CMB kinetic theory.

JIE XIAO,
L^p-Uncertainty Principle from Fractional Schrödinger Equation

Starting with the fractional Schrödinger equation and Fourier analysis, this talk presents a new L^p-uncertainty principle for the positively-ordered Laplace pair $\left\{(-\Delta)^{\frac{\alpha}{2}}, (-\Delta)^{\frac{\beta}{2}}\right\}$.

DEPING YE, Memorial University
The L_p Brunn-Minkowski theory for C-coconvex sets

Recently, Schneider built up the Brunn-Minkowski theory for C-coconvex sets. Unlike convex bodies which are compact convex sets with nonempty interiors in $\mathbb{R}^n$, C-coconvex sets are non-compact convex sets contained in some given pointed closed convex cone in $\mathbb{R}^n$ (a cone with vertex at the origin and having nonempty interior). Schneider obtained many fundamental results on C-coconvex sets, such as the existence of the solutions to the Minkowski problem and the log-Minkowski problem for C-conconvex sets.

In this talk, I will discuss how an L_p Brunn-Minkowski theory can be developed. In particular, I will explain the L_p addition of C-coconvex sets and the related variational formula in terms of volume. I will present the L_p Brunn-Minkowski and L_p Minkowski inequalities for C-coconvex sets. I will also discuss the L_p Minkowski problem for C-coconvex sets (including the existence and uniqueness of its solutions). The case when $p = 0$ is of particular interest, because the log-Brunn-Minkowski and log-Minkowski inequalities can be proved. These results can be applied to the log-Minkowski problem for C-coconvex sets to obtain the uniqueness of solutions to this problem–hence solved an open problem raised by Schneider. Note that the log-Brunn-Minkowski and log-Minkowski inequalities (hence the uniqueness of solutions to the log-Minkowski problem for convex bodies) are still quite open.

EVANGELIE ZACHOS, Stanford University

JOEY ZOU, UC Santa Cruz

The Travel Time Tomography Problem for Transversely Isotropic Elastic Media

I will discuss the travel time tomography problem for elastic media in the transversely isotropic setting. The mathematical study of this problem relates to X-ray tomography and boundary rigidity problems studied by de Hoop, Stefanov, Uhlmann, Vasy, et al., which reduce the inverse problems to the microlocal analysis of certain operators obtained from a pseudo-linearization argument. In the previous works, the authors made strong use of the scattering pseudodifferential calculus, particularly using the inversion theory of elliptic scattering operators. However, in the current setting the analysis is more subtle, as the operators obtained are somewhat degenerate (they resemble parabolic operators in a particular sense, rather than elliptic operators in previous works). In this talk, I will explain the pseudolinearization argument and the qualitative features of the operators obtained, as well as the analysis required to accommodate the slightly more degenerate operators.