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Wronskians, total positivity, and real Schubert calculus

The totally positive flag variety is the subset of the complete flag variety $Fl(n)$ where all Plücker coordinates are positive. By viewing a complete flag as a sequence of subspaces of polynomials of degree at most $n-1$, we can associate a sequence of Wronskian polynomials to it. I will present a new characterization of the totally positive flag variety in terms of Wronskians, and explain how it sheds light on conjectures in the real Schubert calculus of Grassmannians. In particular, a conjecture of Eremenko (2015) is equivalent to the following conjecture: if V is a finite-dimensional subspace of polynomials such that all complex zeros of the Wronskian of V are real and negative, then all Plücker coordinates of V are positive. This conjecture is a totally positive strengthening of a result of Mukhin, Tarasov, and Varchenko (2009), and can be reformulated as saying that all complex solutions to a certain family of Schubert problems in the Grassmannian are real and totally positive.