Approximation Theory in Function Spaces
Théorie de l'approximation dans les espaces de fonctions
(Org: **Javad Mashreghi** (Laval) and/et **Pierre-Olivier Parise** (University of Hawai'i at Manoa))

KELLY BICKEL, Bucknell University
Bounded Rational Functions on the Bidisk

Significant information is known about two-variable functions that are both rational and inner on the bidisk. Known results address both their general structure as well as their behavior and regularity properties near boundary singularities. This talk will discuss how to use a local theory of stable polynomials to study and partially extend these results about rational inner functions to the more general class of bounded rational functions on the bidisk. This is joint work with Greg Knese, James Pascoe, and Alan Sola.

LUDOVICK BOUTHAT, Université Laval
The Hilbert L-matrix and its generalizations

An L-matrix is an infinite matrix which is defined by a sequence $(a_n)_{n\geq 0}$ of positive real numbers and which is of the form

$$A = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 & \dots \\ a_1 & a_1 & a_2 & a_3 & \dots \\ a_2 & a_2 & a_2 & a_3 & \dots \\ a_3 & a_3 & a_3 & a_3 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

These matrices were studied because of their connection with weighted Dirichlet spaces. In earlier work, we studied the Hilbert L-matrix $A_s = [a_{ij}(s)]$, where $a_{ij}(s) = 1/(\max\{i,j\}+s)$ with $i,j \geq 1$. As a surprising property, we showed that its 2-norm is constant for $s \geq s_0$, where the critical point s_0 was unknown until recently. In this presentation, we will show how this phenomenom arises and we establish that the same property persists for the p-norm of A_s matrices. We will also discuss more general properties of L-matrices.

CHRISTOPHER FELDER, Washington University in St. Louis
Approximating analogues of Blaschke products

In this talk we will introduce analogues of both finite and infinite Blaschke products (as inner functions) in a class of general reproducing kernel Hilbert spaces. We will then discuss the approximation of analogues of infinite Blaschke products with their finite counterparts. Time permitting, we will mention a few open problems. Based on work and discussion with T. Le and R. Cheng.

EMMANUEL FRICAIN, Université de Lille
Orthonormal Polynomial Basis in local Dirichlet spaces

Let $\mathbb{D}$ be the open unit disc in the complex plane, and let $\mathbb{T}$ denote its boundary. For $\zeta \in \mathbb{T}$, the local Dirichlet space $\mathcal{D}_\zeta$ consists of functions f analytic on $\mathbb{D}$ such that

$$\int_{\mathbb{D}} |f'(z)|^2 \, \frac{1-|z|^2}{|\zeta - z|^2} \, dA(z) < \infty,$$

where $dA(z) = dx\, dy$ is the planar Lebesgue measure. These spaces have been the focus of numerous studies, e.g., invariant subspaces for the shift operator, multipliers and Carleson measures, connections to de Branges–Rovnyak spaces,...

In this talk, we provide an explicit orthogonal basis of polynomials for the local Dirichlet space $\mathcal{D}_\zeta$, and study their properties. In particular, the latter implies a new polynomial approximation scheme in local Dirichlet spaces.

This is a joint work with Javad Mashreghi.

ADEM LIMANI, Lund university

Approximation problems in model spaces

The model spaces are the invariant subspaces for the backward shift operator on the Hardy space H^2, where the label "model space" stems from the classical theory of Sz.-Nagy and Foias and says that any contractive and completely non-unitary linear operator on Hilbert space can be modeled by the backward shift on a certain model space. Besides their intrinsic operator theoretical nature, these spaces also enjoy some very subtle function theoretical properties. For instance, a classical theorem on approximations on model spaces by A. Aleksandrov says that functions in a model space which extend continuously to the boundary form a dense subset, despite the fact that in many instances, it is very difficult to construct even a single such function. In this talk, we shall investigate the mechanisms which determine when classes of functions enjoying certain regularity properties on the boundary, form a dense subset in the model spaces. This is based on some joint and recent work with B. Malman (KTH).

BARTOSZ MALMAN, KTH Royal Institute of Technology

Smooth Cauchy transforms and constructive approximations in H(b)

We will discuss how construction of certain smooth Cauchy transforms plays a role in constructive approaches to approximations in de Branges-Rovnyak spaces by functions with nice boundary behavior. In particular work of Sergey Khrushchev from 1978 will be mentioned, and we will discuss how constructive proofs of Khrushchev's theorems can be used to develop algorithms for constructive approximations in some special cases of extreme H(b), the main simplifying assumption will be that the symbol b is outer. The talk is based on joint work with Adem Limani from Lund University.

THOMAS RANSFORD, Université Laval

Weakly multiplicative distributions and weighted Dirichlet spaces

We show that if u is a compactly supported distribution on the complex plane such that, for every pair of entire functions f, g,

$$\langle u, f\overline{g}\rangle = \langle u, f\rangle\langle u, \overline{g}\rangle,$$

then u is supported at a single point. As an application, we complete the classification of all weighted Dirichlet spaces on the unit disk that are de Branges–Rovnyak spaces by showing that, for such spaces, the weight is necessarily a superharmonic function. (Joint work with Javad Mashreghi.)

WILLIAM ROSS, University of Richmond

The square root of the Cesaro operator

This joint with with M. Ptak and J. Mashreghi discusses the square root of the classical Cesaro matrix.

MOHAMMAD SHIRAZI, McGill University

Boundary properties of harmonic functions on starlike domains in $\mathbb{R}^n$

I shall present some results recently obtained (with Paul. M. Gauthier) regarding approximating continuous functions by harmonic functions (in the Carleman sense) on some special domains U of $\mathbb{R}^n, n > 1$. In particular, an approximation result on strictly starlike (with respect to the origin and not necessarily bounded) domains in $\mathbb{R}^n$, shall be presented.

It will be shown that the approximation gets better as one moves to the boundary via a subset F whose projection on $\mathbf{S}^{n-1}$ (the unit sphere in $\mathbb{R}^n$) is an F_σ polar set.

ALAN SOLA, Stockholm University
Optimal approximants in the ball and the bidisk: a case study

Reporting on joint work with Meredith Sargent (Manitoba), I will discuss optimal approximants to simple polynomial target functions in function spaces in the ball and the bidisk, respectively. In the case of the ball, a concrete formula for approximating polynomials can be found, but in the bidisk, these polynomials appear to be more mysterious.

MAHISHANKA WITHANACHCHI, Laval University
Approximation by Polynomials in Weighted Dirichlet Spaces

We calculate the exact norm of the partial sum operator S_n for different norms on weighted Dirichlet spaces $\mathcal{D}_w$. We also show some connections to L-matrices with complex entries.