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Orthonormal Polynomial Basis in local Dirichlet spaces

Let $\mathbb{D}$ be the open unit disc in the complex plane, and let $\mathbb{T}$ denote its boundary. For $\zeta \in \mathbb{T}$, the local Dirichlet space $\mathcal{D}_\zeta$ consists of functions f analytic on $\mathbb{D}$ such that

$$\int_{\mathbb{D}} |f'(z)|^2 \, \frac{1 - |z|^2}{|\zeta - z|^2} \, dA(z) < \infty,$$

where $dA(z) = dx \, dy$ is the planar Lebesgue measure. These spaces have been the focus of numerous studies, e.g., invariant subspaces for the shift operator, multipliers and Carleson measures, connections to de Branges–Rovnyak spaces,…

In this talk, we provide an explicit orthogonal basis of polynomials for the local Dirichlet space $\mathcal{D}_\zeta$, and study their properties. In particular, the latter implies a new polynomial approximation scheme in local Dirichlet spaces.

This is a joint work with Javad Mashreghi.