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A closed-form multigrid smoothing factor for an additive Vanka-type smoother applied to the Poisson equation

We consider an additive Vanka-type smoother for the Poisson equation discretized by the standard finite difference centered scheme. Using local Fourier analysis, we derive analytical formulas for the optimal smoothing factors for two types of smoothers, called vertex-wise and element-wise Vanka smoothers, and present the corresponding stencils. Interestingly, in one dimension the element-wise Vanka smoother is equivalent to the scaled mass operator obtained from the linear finite element method, and in two dimensions the element-wise Vanka smoother is equivalent to the scaled mass operator discretized by bilinear finite element method plus a scaled identity operator. Based on these discoveries, the mass matrix obtained from finite element method can be used as an approximation to the inverse of the Laplacian, and the resulting mass-based relaxation scheme features small smoothing factors in one, two, and three dimensions. Advantages of the mass operator are that the operator is sparse and well conditioned, and the computational cost of the relaxation scheme is only one matrix-vector product; there is no need to compute the inverse of a matrix. These findings may help better understand the efficiency of additive Vanka smoothers and develop fast solvers for numerical solutions of partial differential equations.

This work is joint with Chen Greif.