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Brezis–Van Schaftingen–Yung Formulae in Ball Banach Function Spaces

Let X be a ball Banach function space on $\mathbb{R}^n$. In this talk, under the mild assumption that the Hardy–Littlewood maximal operator is bounded on the associated space X' of X, we show that, for any $f \in C_c^2(\mathbb{R}^n)$,

$$\sup_{\lambda\in(0,\infty)} \lambda \left\| \left| \left\{ y \in \mathbb{R}^n : \ |f(\cdot) - f(y)| > \lambda |\cdot - y|^{\frac{n}{q}+1} \right\} \right|^{\frac{1}{q}} \right\|_X \sim \|\nabla f\|_X$$

with the positive equivalence constants independent of f, where $q \in (0,\infty)$ is an index depending on the space X. Particularly, when $X := L^p(\mathbb{R}^n)$ with $p \in [1,\infty)$, the above estimate holds for any given $q \in [1,p]$, which when $q = p$ is exactly a recent surprising formula of H. Brezis, J. Van Schaftingen, and P.-L. Yung. It also enables us to establish new fractional Sobolev and Gagliardo–Nirenberg inequalities in various function spaces, including Morrey spaces, mixed-norm Lebesgue spaces, variable Lebesgue spaces, weighted Lebesgue spaces, Orlicz spaces.