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Localized Spot Patterns for Reaction-Diffusion Systems in 3-D

Localized spot patterns, where one or more solution components concentrates at certain points in the domain, are a common class of localized pattern for reaction-diffusion systems, and they arise in a wide range of modeling scenarios. Although there is a rather well-developed theoretical understanding for this class of localized pattern in 1-D and 2-D, a theoretical study of such patterns in a 3-D setting is, largely, a new frontier. We present some new results for the existence, linear stability, and dynamics of such localized patterns for the 3-D Gierer Meinhardt model. Depending on the parameter range, spot patterns can undergo competition instabilities, leading to spot-annihilation events, or shape-deforming instabilities triggering spot self-replication events. In the absence of these instabilities, the spot locations evolve slowly towards their equilibrium locations according to an ODE gradient flow, which is determined by a discrete energy involving the reduced-wave Green's function. The central role of a certain core problem, which characterizes the profile of a localized spot, on the solution behavior is emphasized. Open problems for localization on higher co-dimension structures, such as stripes and filaments, are discussed.