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Computation of parabolic cylinder functions with complex argument

Parabolic cylinder functions have many mathematical and physical applications. Here we study the function $U(a, z)$, which is characterised as being recessive (exponentially small) at $z = +\infty$. Algorithms for its evaluation for real z are well-established. The case z complex has important applications in asymptotic solutions of differential equations having two turning points in the complex plane, which is a planned further study.

We consider the computation of $U(a, z)$ for unrestricted (unbounded) complex z, with the parameter $a \in [0, \infty)$. Methods for its fast computation are constructed at close to double precision accuracy. To do so, uniform asymptotic expansions for large a with z unrestricted are used, and for the other (small to moderate) values of a we employ contour integral representations, small and large z expansions, along with connection formulas.

This is joint work with Ampero Gil and Javier Segura, Universidad de Cantabria, Santander, Spain.