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A Mathieu function boundary spectral method for acoustic scattering

Many problems in fluid dynamics and acoustics are modelled by singular integral equations with complicated boundary conditions (BCs). This talk considers 2D Helmholtz scattering off (multiple finite) plates, with a focus on BCs ranging from linear models of variable elasticity (fourth-order ODEs), impedance and porosity, to non-linear inertial corrections. A boundary spectral collocation method using Mathieu functions is developed to solve these systems. The method is accurate and flexible for a wide range of frequencies and different BCs, and can robustly compute expansions in tens of thousands of Mathieu functions. As well as discussing numerical analysis aspects, I will demonstrate applications to acoustic black holes, reduction of aerofoil-turbulence interaction noise, and the importance of non-linear corrections for accurately predicting the noise generated by metal foam-like materials. More generally, a goal of this talk is to demonstrate that modern spectral methods can be used in a simple and effective manner for contemporary problems of acoustic scattering, with pointers to ongoing problems.

[1] Colbrook, M.J., Kisil, A.V. "A Mathieu function boundary spectral method for scattering by multiple variable poro-elastic plates, with applications to metamaterials and acoustics." Proceedings of the Royal Society A (2020)

[2] Ayton, L.J., Colbrook, M.J., Geyer, T.F., Paruchuri, C., Sarradj, E. "Reducing aerofoil-turbulence interaction noise through chordwise-varying porosity." JFM (2020)

[3] Colbrook, M.J., Priddin, M.J. "Fast and spectrally accurate numerical methods for perforated screens." IMA Journal of Applied Mathematics (2020)

[4] Colbrook, M.J., Ayton, L.J. "Do we need non-linear corrections? On the boundary Forchheimer equation in acoustic scattering." Submitted