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Defining the spectral position of a Neumann domain

A Laplacian eigenfunction on a Riemannian surface provides a natural partition into Neumann domains—open subsets on which the function satisfies Neumann boundary conditions. These are a natural analogue of nodal domains, on which the eigenfunction satisfies Dirichlet boundary conditions, but their analysis ends up being much more involved.

In this talk I will explain why, on a given Neumann domain, the Neumann Laplacian has compact resolvent (and hence discrete spectrum), and the restricted eigenfunction is an eigenfunction of the Neumann Laplacian. The difficulty in proving these results is that the boundary of a Neumann domain may have cusps and cracks, and hence is not necessarily continuous, so standard density and compactness results for Sobolev spaces are not available. This problem can be overcome using special geometric properties of the Neumann domain boundary, which is made up of gradient flow lines for the corresponding eigenfunction.

These results allow one to define the spectral position of a Neumann domain. (Unlike a nodal domain, the restricted eigenfunction on a Neumann domain is never the ground state.) Finally, I will present a formula for computing the spectral position using the Dirichlet-to-Neumann map. This is joint work with Ram Band and Sebastian Egger.