KATY CRAIG, University of California, Santa Barbara

A blob method for spatially inhomogeneous degenerate diffusion and applications to sampling and two layer neural networks.

Given a desired target distribution on Euclidean space and an initial guess of that distribution, composed of finitely many samples from Euclidean space, what is the best way to move the locations of the samples so that they more accurately represent the desired distribution? A classical solution to this problem is to allow the samples to evolve according to Langevin dynamics, the stochastic particle method for approximating solutions of the Fokker-Planck equation. In today's talk, I will introduce an alternative deterministic particle method for approximating solutions of a spatially inhomogeneous porous medium equation. This method corresponds exactly to the mean-field dynamics of training a two layer neural network for a radial basis function activation function. We prove that, as the number of samples increases and the variance of the radial basis function goes to zero, the particle method converges to a bounded entropy solution of the porous medium equation. As a consequence, we both obtain a novel method for sampling probability distributions as well as insight into the dynamics of training two layer neural networks in the mean field regime, including conditions on which the limiting energy is strongly convex. This is joint work with Karthik Elamvazhuthi (UCLA), Matt Haberland (Cal Poly), and Olga Turanova (Michigan State).