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**Operator algebras, (semi)groups, and dynamics**  
**Algèbres d'opérateur, (semi)groupes et dynamiques**  
(Org: **Chris Bruce** (Queen Mary University of London) and/et **Marcelo Laca** (Victoria))

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**JOHANNES CHRISTENSEN**, KU Leuven

*A new approach to describing KMS states on  $C^*$ -algebras.*

Studying KMS states for  $C^*$ -dynamical systems is a popular theme in the Operator Algebra community in these years. This is in part motivated by the natural interpretation of KMS states in models in physics and in part because KMS states is a powerful tool for building bridges between seemingly unrelated areas of mathematics and uncovering interesting structural properties of  $C^*$ -dynamical systems.

In this talk I will present a new approach to describing KMS states for  $C^*$ -dynamical systems that admit a certain kind of nicely behaved subalgebra of the fixed-point algebra. I will then explain how this description of KMS states provides a new interpretation of a celebrated theorem by Sergey Neshveyev, and how it relates to recent results of Ursu on traces on crossed products.

The results I will present in this talk is joint work with Klaus Thomsen.

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**KRISTIN COURTNEY**, WWU Muenster

*$C^*$ -structure on images of completely positive order zero maps*

A completely positive (cp) map is called order zero when it preserves orthogonality. Such maps enjoy a rich structure, which has made them a key component of completely positive approximations of nuclear  $C^*$ -algebras. Motivated by generalized inductive limits arising from such cp approximations, we consider the structure of the image of a cp order zero map. It turns out that there are a few key properties of a self-adjoint subspace of a  $C^*$ -algebra that characterize when it is the image of a cp order zero map and, moreover, allow us to build a  $C^*$ -structure on that subspace. This is joint work with Wilhelm Winter.

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**TYRONE CRISP**, University of Maine

*An imprimitivity theorem for Hilbert modules*

Mackey's imprimitivity theorem identifies those unitary representations of a group  $G$  that are induced from a representation of a subgroup  $H$ : the induced representations are precisely those that carry a compatible representation of the  $C^*$ -algebra  $C_0(G/H)$ . Rieffel later put this result into the broader context of induced representations of  $C^*$ -algebras: induced representations can in general be characterised by the existence of a compatible representation of an auxiliary  $C^*$ -algebra.

In this talk I shall discuss the related problem of recognising induced Hilbert  $C^*$ -modules. I shall explain why the natural auxiliary object entering into the characterisation of induced modules is a kind of  $C^*$ -coalgebra, rather than a  $C^*$ -algebra; and I will describe two examples in which these somewhat abstract co-algebraic objects can be put into a more familiar  $C^*$ -algebraic form.

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**ANNA DUWENIG**, University of Wollongong

*Cartan subalgebras for non-principal twisted groupoid  $C^*$ -algebras*

The reduced  $C^*$ -algebra of a topologically principal twisted groupoid has a canonical Cartan subalgebra: functions on its unit space. The remarkable Weyl construction, due to Renault, asserts the converse: If a  $C^*$ -algebra  $A$  admits a Cartan subalgebra, there exists such a groupoid whose  $C^*$ -algebra is isomorphic to  $A$  in a Cartan-preserving way. In this talk, I will present on joint work with Gillaspy, Norton, Reznikoff, and Wright, in which we identified subgroupoids of (not necessarily topologically principal) groupoids that give rise to Cartan subalgebras. If time permits, I will further give an explicit description of the associated Weyl groupoid and twist, which is based on further joint work with Gillaspy and Norton.

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**KARI EIFLER**, Texas A&M University  
*Non-local games and quantum metric spaces*

We will look at quantum metric spaces, which are a non-commutative analogue of finite metric spaces. Banica has defined the quantum symmetry group of a finite metric space, and I will talk about how to capture Banica's definition using the Weaver-Kupperberg framework of quantum metric spaces. I will also connect this extension to the theory of non-local games.

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**JAMIE GABE**, University of Southern Denmark  
*Classification of embeddings*

I will survey some recent developments in the classification theory of simple, nuclear  $C^*$ -algebras with an emphasis on a new approach to the main classification theorem. The proof goes through classification of embeddings of  $C^*$ -algebras. This is joint work with José Carrión, Chris Schafhauser, Aaron Tikuisis, and Stuart White.

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**ELIZABETH GILLASPY**, University of Montana  
*Homotopy of product systems, and  $K$ -theory for higher-rank graphs*

One can model the  $C^*$ -algebra of a higher-rank graph ( $k$ -graph) via a product system, which is a higher-dimensional version of a  $C^*$ -correspondence. Just as for the Cuntz–Pimsner algebra associated to a  $C^*$ -correspondence, there is a 6-term exact sequence for the  $K$ -theory of the Cuntz–Nica–Pimsner algebra of a product system. This talk will present joint work with J. Fletcher and A. Sims, in which we establish the compatibility of this 6-term exact sequence with the new notion of a homotopy of product systems, and discuss the applications to higher-rank graphs. Our results imply that certain questions about the  $K$ -theory of  $k$ -graph  $C^*$ -algebras reduce to questions about the path-connectedness of certain spaces of matrices.

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**BEN HAYES**, University of Virginia  
*A random matrix approach to the Peterson-Thom conjecture*

The Peterson-Thom conjecture asserts that any diffuse, amenable subalgebra of a free group factor is contained in a unique maximal amenable subalgebra. This conjecture is motivated by related results in Popa's deformation/rigidity theory and Peterson-Thom's results on  $L^2$ -Betti numbers. We present an approach to this conjecture in terms of so-called strong convergence of random matrices by formulating a conjecture which is a natural generalization of the Haagerup-Thorbjornsen theorem whose validity would imply the Peterson-Thom conjecture. This random matrix conjecture is related to recent work of Collins-Guionnet-Parraud. This talk will be accessible to  $C^*$ -algebraists. I promise.

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**MATTHEW KENNEDY**, University of Waterloo  
*Amenability, proximality and higher order syndeticity*

I will present new descriptions of some universal flows associated to a discrete group, obtained using what we view as a kind of "topological Furstenberg correspondence." The descriptions are algebraic and relatively concrete, involving subsets of the group satisfying a higher order notion of syndeticity. We utilize them to establish new necessary and sufficient conditions for strong amenability and amenability. Furthermore, utilizing similar techniques, we obtain a characterization of "dense orbit sets," answering a question of Glasner, Tsankov, Weiss and Zucker. Throughout the talk, I will discuss connections to operator algebras.

This is joint work with Sven Raum and Guy Salomon.

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**NADIA LARSEN**, University of Oslo, Norway  
*Equilibrium states on  $C^*$ -algebras of right lcm monoids*

Left cancellative monoids with right lcm's (least common multiples) for every pair of elements with a common right multiple cover a variety of examples, such as Zappa-Szep products from self-similar actions of groups, and several classes of semidirect products. The  $C^*$ -algebras of such monoids that display a generalised scale, namely a particular type of monoid homomorphism into the multiplicative natural numbers, admit a characterisation of all their equilibrium states. I will present some of the key ideas in this characterisation. The talk is based on a joint work with Nathan Brownlowe, Jacqui Ramagge and Nicolai Stammeier.

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**BOYU LI**, University of Victoria

*The Zappa-Szép product of a Fell bundle by a groupoid*

In group theory, the Zappa-Szép product generalizes the semi-direct product. As the semi-direct product is related to the crossed product of operator algebras, we seek to define a Zappa-Szép product analogue. First, we define the notion of the compatible groupoid action on a Fell bundle, which allows us to define the Zappa-Szép product of a Fell bundle by a groupoid. We show that this product is a Fell bundle over the Zappa-Szép product of the underlying groupoids. We then show that the representation of the Zappa-Szép product Fell bundle is related to the notion of covariant representations. Finally, we briefly discuss some basic properties of its  $C^*$ -algebra. This is a joint work with Anna Duwenig.

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**XIN LI**, University of Glasgow

*K-theory for semigroup  $C^*$ -algebras and partial crossed products*

We present a K-theory formula for a class of inverse semigroup  $C^*$ -algebras. As a special case, we discuss  $C^*$ -algebras generated by left regular representations of semigroups, and illustrate the main theorem and its applications with concrete examples.

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**HUNG-CHANG LIAO**, University of Ottawa

*Almost finiteness, comparison, and tracial Z-stability*

Inspired by Kerr's work on topological dynamics, we define tracial Z-stability for sub- $C^*$ -algebras. We will discuss how it is related to dynamical properties such as almost finiteness and comparison. The talk is based on a joint work with Aaron Tikuisis.

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**CAMILA FABRE SEHNEM**, Victoria University of Wellington

*Nuclearity for partial crossed products by exact discrete groups*

Important classes of  $C^*$ -algebras can be described as partial crossed products. Even though a partial action of a discrete group on a  $C^*$ -algebra is in an appropriate sense always equivalent to a global action, the commutativity of the underlying  $C^*$ -algebra may be lost under this correspondence. I will talk about a joint work with A. Buss and D. Ferraro, in which we generalise a result by Matsumura for ordinary actions by showing that the partial crossed product of a commutative  $C^*$ -algebra by an exact discrete group is nuclear whenever the full and reduced partial crossed products coincide. We apply our results to show that the reduced semigroup  $C^*$ -algebra  $C_\lambda^*(P)$  of a submonoid of an exact discrete group is nuclear if the left regular representation on  $\ell^2(P)$  is an isomorphism between the full and reduced  $C^*$ -algebras.

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**KAREN STRUNG**, Institute of Mathematics, Czech Academy of Sciences

*Constructions in minimal amenable dynamics and applications to classification of  $C^*$ -algebras.*

What abelian groups can arise as the  $K$ -theory of  $C^*$ -algebras arising from minimal dynamical systems? In joint work with Robin Deeley and Ian Putnam, we completely characterize the  $K$ -theory of the crossed product of a space  $X$  with finitely generated  $K$ -theory by an action of the integers and show that crossed products by a minimal homeomorphism exhaust the range of these possible  $K$ -theories. We also investigate the  $K$ -theory and the Elliott invariants of orbit-breaking algebras. We show that given arbitrary countable abelian groups  $G_0$  and  $G_1$  and any Choquet simplex  $\Delta$  with finitely many extreme points,

we can find a minimal orbit-breaking relation such that the associated  $C^*$ -algebra has  $K$ -theory given by this pair of groups and tracial state space affinely homeomorphic to  $\Delta$ . These results have important applications to the Elliott classification program for  $C^*$ -algebras. In particular, we make a step towards determining the range of the Elliott invariant of the  $C^*$ -algebras associated to étale equivalence relations.

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**TAKUYA TAKEISHI**, Kyoto Institute of Technology

*Partition functions as  $C^*$ -dynamical invariants and actions of congruence monoids*

$C^*$ -algebras of  $ax + b$ -semigroups of congruence monoids  $C_\lambda^*(R \rtimes R_{m,\Gamma})$  are introduced by C. Bruce, which behaves similarly to the  $C^*$ -algebras examined by Cuntz–Deninger–Laca. Both kinds of algebras have canonical time evolutions, and have similar phase transition phenomena. In this talk, we determine the partition functions and associated representations of  $(C_\lambda^*(R \rtimes R_{m,\Gamma}), \sigma)$ , inspired by the construction of the representations of Cuntz–Deninger–Laca. As a consequence, we recover several number theoretic invariants from those  $C^*$ -dynamical systems. In the case of  $(C_\lambda^*(R \rtimes R^\times), \sigma)$ , we in fact obtain slightly different partition functions from those suggested in the work of Cuntz–Deninger–Laca. This is a joint work with C. Bruce and M. Laca.

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**AARON TIKUISIS**, University of Ottawa

*Classification of embeddings II*

In a sequel to Jamie’s talk, I will further discuss recent developments in the classification of nuclear simple  $C^*$ -algebras and full embeddings. This is joint work with José Carrión, Jamie Gabe, Chris Schafhauser, and Stuart White.

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**DAN URSU**, University of Waterloo

*Characterizing traces on crossed products of noncommutative  $C^*$ -algebras*

Given a discrete group  $G$  acting on a unital  $C^*$ -algebra  $A$ , a crossed product is a  $C^*$ -algebra containing a copy of  $G$  (as unitaries) and  $A$ , where the action of  $G$  on  $A$  is now inner. This comes in two main flavours - the universal crossed product  $A \rtimes G$  and the reduced crossed product  $A \rtimes_r G$ .

We give complete descriptions of the tracial states on both the universal and reduced crossed products. In particular, we also answer the question of when the tracial states are in canonical bijection with the  $G$ -invariant tracial states on  $A$ . This generalizes the unique trace property for discrete groups. The analysis simplifies greatly in various cases, such as in the case of FC groups, more so for abelian groups, and even more so in the case of  $G = \mathbb{Z}$ . In other cases, we obtain previously known results, for example when  $A = C(X)$ .

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**MARIA GRAZIA VIOLA**, Lakehead University

*Regularity properties of Cuntz-Pimsner algebras associated to  $C^*$ -correspondences over commutative  $C^*$ -algebras*

We discuss the structural properties of Cuntz-Pimsner algebras arising from full, minimal, non-periodic, and finitely generated projective  $C^*$ -correspondence over commutative  $C^*$ -algebras. A large class of examples is obtained considering the set  $\Gamma(V, \alpha)$  of continuous sections of a complex vector bundle on a compact metric space  $X$ , where left multiplication is given by a twist by a minimal homeomorphism  $\alpha: X \rightarrow X$ .

Cuntz-Pimsner algebras are generalization of both Cuntz-Krieger algebras and crossed products by the integers. In the case of crossed products by minimal homeomorphisms, the orbit breaking subalgebra, defined by I. Putnam, is a large subalgebra in the sense of N. C. Phillips. We show that the Cuntz-Pimsner algebra  $\mathcal{O}(\Gamma(V, \alpha))$  also contains a large subalgebras, at least for a large class of  $C^*$ -correspondences. We will discuss some properties that  $\mathcal{O}(\Gamma(V, \alpha))$  and/or its large subalgebra have, focusing on properties needed for classification by the Elliott invariant.

This is joint work with M. S. Adamo, D. Archey, M. Forough, M. Georgescu, J. A. Jeong, and K. Strung.

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**DILIAN YANG**, University of Windsor  
*Zappa-Szép Actions of Groups on Product Systems*

Let  $G$  be a group and  $X$  be a product system over a semigroup  $P$ . Suppose  $G$  has a left action on  $P$  and  $P$  has a right action on  $G$ , so that one can form a Zappa-Szép product  $P \bowtie G$ . We define a *Zappa-Szép action* of  $G$  on  $X$ , roughly speaking, to be a collection of functions on  $X$ , which is compatible with both actions from  $P \bowtie G$ . For a given Zappa-Szép action of  $G$  on  $X$ , we construct a new product system  $X \bowtie G$  over  $P \bowtie G$ , which is called the Zappa-Szép product of  $X$  by  $G$ . Then we associate  $X \bowtie G$  some universal  $C^*$ -algebras, and show some Hao-Ng type isomorphisms. A special case of interest is when the action is homogenous.

This is ongoing joint work with Boyu Li.