Homotopy Theory
Théorie de l'Homotopie

(Org: **Agnès Beaudry** (University of Colorado Boulder), **Martin Frankland** (University of Regina) and/et **Donald Stanley** (University of Regina))

KATHARINE ADAMYK, Western University

Lifting A(1)-Modules

The Steenrod algebra, $\mathcal{A}$, arises topologically as the algebra of stable operations on cohomology. For any nonnegative integer n, we consider $\mathcal{A}(n)$, a particular subalgebra of $\mathcal{A}$. Given an $\mathcal{A}(n)$-module, M, for some n, we can ask whether it lifts to a module over $\mathcal{A}$. (That is, whether there exists any $\mathcal{A}$-module whose underlying $\mathcal{A}(n)$-module is M.)

In this talk, we will focus on lifting $\mathcal{A}(1)$-modules. Some obstructions to these lifting problems are detected via a spectral sequence that computes localized Ext groups. The computation of this spectral sequence can be simplified by a classification theorem for a particular class of $\mathcal{A}(1)$-modules.

STEVEN AMELOTTE, University of Rochester

The homotopy type of the fibre of the p^{th} power map on loop spaces of spheres

The problem of decomposing the fibre of the p^{th} power map on loop spaces of spheres into a product of indecomposable factors has a long history with relations to the homotopy exponents of spheres, Kervaire invariant one classes, the Kahn–Priddy theorem and classifying spaces for the fibre of the double suspension. In this talk I will discuss the remaining unresolved cases and outline a proof that, for odd primes p, the decomposition problem for $\Omega S^{2n+1}\{p\}$ is equivalent to the p-primary Kervaire invariant problem.

KRISTINE BAUER, University of Calgary

Operads of functors with derivatives

The Goodwillie functor calculus tower is an approximation of a homotopy functor which resembles the Taylor series approximation of a function in ordinary calculus. In 2004-05, several authors observed that the homogeneous layers of certain Goodwillie towers form an operad. Ching first observed that the identity functor of spaces has this behaviour. McCarthy and Minasian observed similar operads for functors of operad algebras which are monoids.

In 2011, Cockett and Seely used the notion of categorical differentiation to construct a Faa di Bruno formula, and an associated category Faa(X), which encapsulates the higher order chain rules for derivatives.

The goal of this talk is to explain why one should expect an operad structure associated to the layers of functor calculus towers from the perspective of ordinary calculus. This is an update of a program of research by Johnson, Yeakel and I which shows that the functor associating a functor of abelian categories to its sequence of derivatives is monoidal. This explains the operads arising from functor calculus towers as a consequences of differentiation.

MARZIEH BAYEH, University of Ottawa

Higher Equivariant and Invariant Topological Complexities

The topological complexity was introduced by Farber to estimate the complexity of the configuration space of a robot or a mechanical system.

Later, Rudyak introduced a series of invariants $\{TC_n(X)\}$, called the higher topological complexity, which is related to a motion planning algorithm with n points as the input (in addition to the initial and terminal states of the robot, some intermediate states are given as well).

If the configuration space admits an action of a topological group G (for example having a symmetry on the mechanical system or its configuration space), then it is worth considering a motion planning algorithm that is compatible with the action. There are different approaches to define an equivariant version of topological complexity.

In this talk we will consider two of those approaches and discuss a generalization of each invariant in the realm of higher topological complexity.

BRANDON DOHERTY, University of Western Ontario
Cubical models of (infinity,1)-categories

We discuss the construction of a new model structure on the category of cubical sets with connections whose cofibrations are the monomorphisms and whose fibrant objects are defined by the right lifting property with respect to inner open boxes, the cubical analogue of inner horns. We also discuss the proof that this model structure is Quillen equivalent to the Joyal model structure on simplicial sets via the triangulation functor. This talk is based on joint work with Chris Kapulkin, Zachery Lindsey, and Christian Sattler, arXiv:2005.04853.

RACHEL HARDEMAN, University of Calgary
A Search for Model Structure for A-Homotopy Theory

A-homotopy theory is a proposed homotopy theory for simple graphs. The foundations of this theory were first developed by Ron Aktin in the 1970s and further developed by Barcelo et. al. in the early 2000s. The literature has primarily focused on defining A-homotopy groups and finding ways to compute them. In this talk, I will give a brief introduction to A-homotopy theory with some examples and discuss the search for a model structure on the category GPH that would correspond to this theory. If found, this model structure would give us all the machinery to work in A-homotopy theory that has yet to be developed.

SACHA IKONICOFF, University of Calgary
Unstable algebras over an operad

The aim of this talk is the study of algebraic operations that naturally appear on classical unstable modules over the Steenrod algebra, especially (but not exclusively) those modules that do not come from topological spaces, such as Brown-Gitler modules or Carlsson modules. We will show how the theory of algebraic operads fits into this framework. In characteristic 2, we will define a notion of unstable algebra over an operad relatively to a commutative operation of the operad. Under some hypotheses on the operad $\mathcal{P}$, on the operation $\star \in \mathcal{P}(2)^{\mathfrak{S}_2}$, and on the unstable module M, we will identify the free $\star$-unstable $\mathcal{P}$-algebra generated by M to a free $\mathcal{P}$-algebra. This will allow us to recollect some results of Steenrod-Epstein and Serre regarding the cohomology of Eilenberg-MacLane spaces, as well as a result of D. Davis on the Carlsson module of weight 1.

SANDER KUPERS, University of Toronto
The rational homotopy type of certain diffeomorphism groups

This talk concerns joint work with Oscar Randal-Williams which aims to understand the topological group of diffeomorphisms of an even-dimensional disc, fixing its boundary pointwise. This is one of the most important objects in geometric topology. I will explain how we computed many of its rational homotopy groups, and describe what we expect the final answer to be.

IVAN LIMONCHENKO, University of Toronto
On homotopy theory of polyhedral products with Golod face rings

In 1950s J.-P.Serre proved that Poincaré series of a commutative local Noetherian ring is bounded by a certain rational function depending on the Betti numbers of the Koszul complex and the minimal number of generators in the maximal ideal. In 1962 E.S.Golod showed that Serre's inequality turns into equality if and only if multiplication and all Massey products in Koszul homology of a local ring are trivial. J.Backelin proved in 1982 that Poincaré series of monomial rings are rational; among monomial rings there is the well-known class of Stanley-Reisner rings (or, face rings) of simplicial complexes.

In this talk we will discuss homotopy theory of polyhedral products over simplicial complexes having Golod face rings over fields. We will describe this class of Stanley-Reisner rings in terms of their Poincaré series, Koszul homology, and the loop homology algebra structure of moment-angle-complexes. Much more can be said if only flag simplicial complexes are considered. We will see how the methods and objects of toric topology allow us to obtain topological interpretations of the algebraic properties of Poincaré series and Koszul homology of Stanley-Reisner rings as well as to get new results.

The talk is partially based on an ongoing research project joint with Kouyemon Iriye, Daisuke Kishimoto, and Taras Panov.

NINY ARCILA MAYA, University of British Columbia
Decomposition of topological Azumaya algebras with involution

A topological Azumaya algebra with an involution is a topological generalization of the concept of a central simple algebra with an involution. We give conditions for positive integers m and n and the space X such that a topological Azumaya algebra with an involution of degree mn over X can be decomposed as the tensor product of topological Azumaya algebras with involution of degrees m and n.

NICHOLAS MEADOWS, Carleton University
Spectral Sequences in $(\infty, 1)$-categories

Many spectral sequences in algebraic topology and other areas, can be realized as the spectral sequence associated to a (co)simplicial space. Examples include the Eilenberg-Moore and Adams spectral sequences. In this talk, we will explain how to set up the spectral sequence associated to a simplicial object in an $(\infty, 1)$-category, in a model-independent manner. We will also show how the differentials and filtration of the spectral sequence can be described in terms of the combinatorics of the ambient $(\infty, 1)$-category.

Joint work with D. Blanc.

APURVA NAKADE, University of Western Ontario
Discrete Chern-Simons via 2-group bundles on elliptic curves

Freed-Quinn define a model for 2+1 Chern-Simons theory with a finite gauge group G by constructing a particular line bundle $\mathcal{L}$ on the moduli space of flat principal G bundles over a genus g surface X. In this talk, I will explain their construction and then show that $\mathcal{L}$ is naturally a 2-group bundle over X, where a 2-group can be thought of as a categorified version of a group with a weaker notion of associativity. Our results provide a concrete example of a mathematical physics phenomenon that can be most naturally described using higher categorical language. This talk is based on joint work with D. Berwick-Evans, E. Cliff, L. Murray, and E. Phillips.

KATE POIRIER, City University of New York - NYCCT
Polyhedra for V-infinity algebras, string topology, and moduli spaces

Where associahedra are polyhedra that organize operations and relations in an A_∞ algebra, assocoipahedra are polyhedra that organize operations and relations in a V_∞ algebra, a homotopy version of an associative algebra that has a compatible co-inner product. Assocoipahedra appear in the study of spaces of string topology operations—both on the chains or homology of the loop space of a closed, oriented manifold (the topological side) and on the Hochschild cochains or cohomology of a V_∞ algebra (the algebraic side). We describe the role assocoipahedra play on both sides and present progress on a conjecture relating these spaces of operations to the moduli space of Riemann surfaces.

DORETTE PRONK, Dalhousie University

Three approaches toward orbifold mapping objects

We consider orbispaces as proper étale groupoids (also called *orbigroupoids*) in the category of locally compact, paracompact Hausdorff spaces. When defined this way, two groupoids represent equivalent orbispaces precisely when they are Morita equivalent. So we consider the bicategory of fractions with respect to Morita equivalences. For orbispaces G and H we can then consider the mapping groupoid $\mathbf{OMap}(G, H)$ of generalized maps and equivalences classes of 2-cell diagrams. The question I want to address is how to define a topology on these mapping groupoids to obtain mapping objects for this bicategory. I will approach this question from three different directions:

1. When the orbifold G is compact, we can define a topology on $\mathbf{OMap}(G, H)$ so that

$$\mathbf{Orbispaces}(K \times G, H) \simeq \mathbf{Orbispaces}(K, \mathbf{OMap}(G, H)).$$

2. For any pair of orbigroupoids G, H we can define a topology on $\mathbf{OMap}(G, H)$ so that $\mathbf{Orbispaces}$ has the structure of an enriched bicategory.

3. There is a fibration structure on the category of orbigroupoids with groupoid homomorphisms as defined in [Pronk-Warren]. (This can be derived from work by Colman and Costoya.) This implies that when G and H are stack groupoids, we may restrict ourselves to groupoid homomorphisms and their usual 2-cells.

In this talk I will discuss the relationships between the topologies obtained in these ways. This is joint work with Laura Scull and Hellen Colman.

[Pronk-Warren] Dorette A. Pronk, Michael A. Warren, Bicategorical fibration structures and stacks, Theory and Applications of Categories, Vol. 29, 2014, No. 29, pp 836-873.

LUIS SCOCCOLA, Michigan State University

Homotopy coherence in applied topology

A persistent object consists of a diagram indexed by the poset of real numbers. The interleaving distance is a natural way of comparing persistent objects, and is used to state and prove that certain algorithms in applied topology are stable to perturbations of the input dataset. For persistent objects of a model category there exist several ways of weakening the interleaving distance in order to make it homotopy-invariant, and comparing these choices requires solving rectification problems that can be approached using tools from homotopy theory. I will discuss positive and negative rectification results recently obtained in joint work with Edoardo Lanari.