DORETTE PRONK, Dalhousie University

Three approaches toward orbifold mapping objects

We consider orbispaces as proper étale groupoids (also called *orbigroupoids*) in the category of locally compact, paracompact Hausdorff spaces. When defined this way, two groupoids represent equivalent orbispaces precisely when they are Morita equivalent. So we consider the bicategory of fractions with respect to Morita equivalences. For orbispaces G and H we can then consider the mapping groupoid $\mathbf{OMap}(G,H)$ of generalized maps and equivalences classes of 2-cell diagrams. The question I want to address is how to define a topology on these mapping groupoids to obtain mapping objects for this bicategory. I will approach this question from three different directions:

1. When the orbifold G is compact, we can define a topology on $\mathbf{OMap}(G,H)$ so that

$$\mathbf{Orbispaces}(K\times G,H)\simeq\mathbf{Orbispaces}(K,\mathbf{OMap}(G,H)).$$

2. For any pair of orbigroupoids G, H we can define a topology on $\mathbf{OMap}(G,H)$ so that $\mathbf{Orbispaces}$ has the structure of an enriched bicategory.

3. There is a fibration structure on the category of orbigroupoids with groupoid homomorphisms as defined in [Pronk-Warren]. (This can be derived from work by Colman and Costoya.) This implies that when G and H are stack groupoids, we may restrict ourselves to groupoid homomorphisms and their usual 2-cells.

In this talk I will discuss the relationships between the topologies obtained in these ways. This is joint work with Laura Scull and Hellen Colman.

[Pronk-Warren] Dorette A. Pronk, Michael A. Warren, Bicategorical fibration structures and stacks, Theory and Applications of Categories, Vol. 29, 2014, No. 29, pp 836-873.