Equidistribution on Arithmetic Manifolds
Équidistributions sur les Variétés Arithémtiques
(Org: **Ilya Khayutin** (Northwestern) and/et **Simon Marshall** (Wisconsin))

LINDSAY DEVER, Bryn Mawr College
Ambient prime geodesic theorems on compact hyperbolic 3-manifolds

The study of hyperbolic 3-manifolds draws deep connections between number theory, geometry, topology, and quantum mechanics. Specifically, the closed geodesics on a manifold are intrinsically related to the eigenvalues of Maass forms via the Selberg trace formula and are parametrized by their length and holonomy. The trace formula for spherical Maass forms can be used to prove the Prime Geodesic Theorem, which provides an asymptotic count of geodesics up to a certain length. In 1999, Sarnak and Wakayama established a count of geodesics by both length and holonomy that shows that holonomies of geodesics of increasing lengths become equidistributed throughout the circle. In this talk, I will present new results including a count of geodesics in shrinking intervals of length and holonomy, which implies effective equidistribution of holonomy.

SHAI EVRA, Princeton
Ramanujan Conjecture and the Density Hypothesis

The Generalized Ramanujan Conjecture (GRC) for GL(n) is a central open problem in modern number theory. Its resolution is known to yield several important applications. For instance, the Ramanujan-Petersson conjecture for GL(2), proven by Deligne, was a key ingredient in the work of Lubotzky-Phillips-Sarnak on Ramanujan graphs. One can also state analogues (Naive) Ramanujan Conjectures (NRC) for other reductive groups. However, in the 70's Kurokawa and Howe-Piatetski-Shapiro proved that the (NRC) fails even for quasi-split classical groups. In the 90's Sarnak-Xue put forth a Density Hypothesis version of the (NRC), which serves as a replacement of the (NRC) in applications. In this talk I will describe a possible approach to proving the Density Hypothesis for definite classical groups, by invoking deep and recent results coming from the Langlands program: The endoscopic classification of automorphic representations of classical groups due to Arthur, and the proof of the Generalized Ramanujan-Petersson Conjecture.

MIKOLAJ FRACZYK, University of Chicago
Density hypothesis in horizontal families

Let G be a real semi simple Lie group with an irreducible unitary representation π. The non-temperedness of π is measured by the real parameter $p(\pi)$ which is defined as the infimum of $p \geq 2$ such that π has non-zero matrix coefficients in $L^p(G)$. Sarnak and Xue conjectured that for any arithmetic lattice $\Gamma \subset G$ and principal congruence subgroup $\Gamma(q) \subset \Gamma$, the multiplicity of π in $L^2(G/\Gamma(q))$ is at most $O(V(q)^{2/p(\pi)+\epsilon})$ where $V(q)$ is the covolume of $\Gamma(q)$. Sarnak and Xue proved this conjecture for $G = SL(2,R), SL(2,\mathbb{C})$. In a a joint work with Gergely Harcos, Peter Maga and Djordje Milicevic we prove bounds of the same quality that hold uniformly for families of pairwise non-commensurable lattices in $G = SL(2,\mathbb{R})^a \times SL(2,\mathbb{C})^b$ given as unit groups of maximal orders of quaternion algebras over number fields ("horizontal families"). I will also discuss how the multiplicity bounds depend on the Archimedean parameters and some possible extensions of our methods.

MATHILDE GERBELLI-GAUTHIER, Centre de Recherches Mathématiques
Limit multiplicity of non-tempered representations and endoscopy.

How fast do Betti numbers grow in a congruence tower of compact arithmetic manifolds? The question can be reformulated in terms of limit multiplicity of representations. If the representation is discrete series, the rate of growth is known to be proportional to the volume of the manifold; otherwise the growth is sub-linear in the volume. Sarnak-Xue have conjectured that bounds on multiplicity growth can be expressed in terms of the failure of representations to be tempered. I will confirm some instances of the Sarnak-Xue conjecture for unitary groups using the fact that some non-tempered representations arise as endoscopic transfer, and give applications to cohomology growth.

ALIREZA SALEHI GOLSEFIDY, UCSD

Two new concepts for compact groups: Spectral independence and local randomness

I will explain two new concepts for compact groups mentioned in the title. Their basic properties and their connections with the FAb property, quasi-randomness, and super-approximation will be outlined. I will present how these ideas help us show that a Borel probability measure m on the product of compact open subgroups of two locally non-isomorphic simple analytic groups has the spectral gap property when its projection to each factor has. (Joint work with Keivan Mallahi-Karai and Amir Mohammadi)

THOMAS HILLE, Northwestern

Bounds for the Least Solution of Homogeneous Quadratic Diophantine Inequalities.

Let Q be a non-degenerate indefinite quadratic form in d variables. In the mid 80's, Margulis proved the Oppenheim conjecture, which states that if $d \geq 3$ and Q is not proportional to a rational form then Q takes values arbitrarily close to zero at integral points. In this talk we will discuss the problem of obtaining bounds for the least integral solution of the Diophantine inequality $|Q[x]| < \epsilon$ for any positive ϵ if $d \geq 5$. We will show how to obtain explicit bounds that are polynomial in ϵ^{-1}, with exponents depending only on the signature of Q or if applicable, the Diophantine properties of Q. This talk is based on joint work with P. Buterus, F. Götze and G. Margulis.

JUNEHYUK JUNG, Brown University

Intersections of geodesics on the modular surface

Let α be a compact geodesic segment in the full modular surface, and let C_D be the union of closed geodesics of discriminant D. I'm going to present a proof that the intersection points $\alpha \cap C_D$ become equidistributed along α as $D \to \infty$. I will then discuss how to make the theorem effective. This is a joint work with Naser Talebizadeh Sardari.

ASAF KATZ, University of Michigan

An application of Margulis' inequality to effective equidistribution

Ratner's celebrated equidistribution theorem states that the trajectory of any point in a homogeneous space under a unipotent flow is getting equidistributed with respect to some algebraic measure. In the case where the action is horospherical, one can deduce an effective equidistribution result by mixing methods, an idea that goes back to Margulis' thesis. When the homogeneous space is non-compact, one needs to impose further "diophantine conditions" over the base point, quantifying some recurrence rates, in order to get a quantified equidistribution result.

In the talk I will discuss certain diophantine conditions, and in particular I will show how a new Margulis' type inequality for translates of horospherical orbits helps verify such conditions, a quantified equidistribution result for a large class of points, akin to the results of A. Strombreggson dealing with SL_2 case. In particular we deduce a fully effective quantitative equidistribution for horospherical trajectories of lattices defined over number fields, without pertaining to the strong subspace theorem.

ALEX KONTOROVICH, Rutgers

Applications of Thin Orbits

We describe some applications of equidistribution of arithmetic manifolds to thin orbits.

ARIE LEVIT, Yale university

Quantitative weak uniform discreteness

I will discuss a quantitative variant of the classical Kazhdan-Margulis theorem generalized to probability measure preserving actions of semisimple groups over local fields. More precisely, the probability that the stabilizer of a random point admits a non-trivial intersection with a small r-neighborhood of the identity is at most br^d, for some explicit constants $b, d > 0$ depending only on the semisimple group in question. Our proof involves some of the original ideas of Kazhdan and Margulis, combined with methods of Margulis functions as well as (C, α)-good functions on varieties. As an application, we present a new unified proof of the fact that all lattices in these groups are weakly cocompact, i.e admit a spectral gap. The talk is based on a preprint joint with Gelander and Margulis.

NICHOLAS MILLER, UC Berkeley
Geodesic submanifolds of hyperbolic manifolds

It is a consequence of the Margulis dichotomy that when an arithmetic hyperbolic manifold contains one totally geodesic hypersurface, it contains infinitely many. Both Reid and McMullen have asked conversely whether the existence of infinitely many geodesic hypersurfaces implies arithmeticity of the corresponding real hyperbolic manifold. In this talk, I will discuss recent results answering this question in the affirmative. This is joint work with Bader, Fisher, and Stover.

AMIR MOHAMMADI, UCSD
Effective results in homogeneous dynamics

Rigidity phenomena in homogeneous dynamics have been extensively studied over the past few decades with several striking results and applications. In this talk we will give an overview of the more recent activities which aim at presenting quantitative versions of some of these strong rigidity results.

WENYU PAN, University of Chicago
Exponential mixing of geodesic flows for geometrically finite hyperbolic manifolds with cusps

Let $\mathbb{H}^n$ be the hyperbolic n-space and Γ be a geometrically finite discrete subgroup in $\mathrm{Isom}_+(\mathbb{H}^n)$ with parabolic elements. In the joint work with Jialun LI, we establish the exponential mixing of the geodesic flow over the unit tangent bundle $T^1(\Gamma\backslash\mathbb{H}^n)$ with respect to the Bowen-Margulis-Sullivan measure. This kind of result is known to have many immediate applications in number theory and geometry, which includes counting closed geodesics and shrinking target problems. Our approach is to construct coding for the geodesic flow and then prove a Dolgopyat-type spectral estimate for the corresponding transfer operator. I will also discuss the application of obtaining a resonance-free region for the resolvent on $\Gamma\backslash\mathbb{H}^n$.

LAM PHAM, Brandeis University
Arithmetic Groups and the Lehmer conjecture

The Lehmer problem (1933), also referred to as the 'Lehmer conjecture', asks whether there is a uniform lower bound on the Mahler measure of algebraic integers which are not roots of unity. Although rooted in number theory, many interesting connections between the Lehmer problem and other fields, including combinatorics in finite fields and geometric group theory. Following his celebrated Arithmeticity Theorem, Margulis conjectured in his book (1991) the uniform discreteness of cocompact lattices in higher rank semisimple Lie groups and observed that it would follow from a weak form of the Lehmer conjecture. We will discuss the equivalence of these conjectures and some refinements. This is based on joint work with François Thilmany.

WILL SAWIN, Columbia University
The mixing conjecture over function fields

Michel and Venkatesh conjectured a generalization of Duke's theorem - that Galois orbits of CM points are equidistributed on a product of two modular curves. Shende and Tsimerman proved a function-field analogue of this conjecture, conditional on a hypothesis, which I verified in recent work. Also recently, Khayutin has proven the original conjecture under some assumptions, and even more recently Blomer and Brumley have proven a weaker form of the conjecture under a different set of assumptions. These proofs all use very different methods and have distinct strengths and weaknesses. I will describe some of the key ideas of the geometric approach of Shende, Tsimerman, and myself.

MATTHEW YOUNG, Texas A&M

Moments and hybrid subconvexity for symmetric-square L-functions

I will discuss some recent work on moment problems for symmetric-square L-functions. One application of this work is a hybrid subconvexity result for these L-functions, and another is a short interval Lindelof-on-average bound. I will also discuss some of the motivation for these problems, which relates these L-functions to the equidistribution of cusp forms on the modular surface. This is joint work with Rizwan Khan.