
Derived Categories and (Non)commutative Algebraic Geometry
Catégories Dérivées et Géométrie Algébrique (Non) Commutative
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DYLAN ALLEGRETTI, University of British Columbia

Stability conditions and cluster varieties

I will describe a construction that associates a 3-Calabi-Yau triangulated category to a quiver with potential. There are two interesting spaces that reflect the structure of this category. The first is a complex manifold parametrizing Bridgeland stability conditions, and the second is a kind of non-separated scheme called a cluster variety. I will describe a theorem that relates these two spaces in a large class of examples arising from triangulated surfaces.

SABIN CAUTIS, UBC

Categorical structure of Coulomb branches of 4D $N=2$ gauge theories

We will discuss the categorical structure of Coulomb branches. For concreteness we focus on the massless case which is just the category of coherent sheaves on the affine Grassmannian (the coherent Satake category).

These categories are conjecturally governed by a cluster algebra structure. We describe a solution of this conjecture in the case of general linear groups and discuss its extension to more general Coulomb branches of 4D $N=2$ gauge theories. This is joint work with Harold Williams.

KATRINA HONIGS, University of Oregon

An obstruction to weak approximation on some Calabi-Yau threefolds

There has been recent interest in whether existence and density of \mathbb{Q} -rational points is preserved under derived equivalence. After giving a short introduction to this question, I will be discussing recent work joint with Hashimoto, Lamarche and Vogt in which we examine \mathbb{Q} -points on a family of derived equivalent Calabi-Yau threefolds. These threefolds were constructed and analyzed in detail as complex varieties by Hosono and Takagi in the context of mirror symmetry. One family of threefolds occurs as a linear section of a double quintic symmetroid, and we are able to give a general condition under which a Brauer class obstructs weak approximation, though it cannot obstruct the existence of \mathbb{Q} -rational points.

COLIN INGALLS, Carleton University

Explicit coverings of families of elliptic surfaces by squares of curves

We show that, for each $n > 0$, there is a family of elliptic surfaces which are covered by the square of a curve of genus $2n + 1$, and whose Hodge structures have an action by $\mathbb{Q}(\sqrt{-n})$. By considering the case $n=3$, we show that one particular family of K3 surfaces are covered by the square of genus 7. Using this, we construct a correspondence between the square of a curve of genus 7 and a general K3 surface in \mathbb{P}^4 with 15 ordinary double points up to isogeny. This gives an explicit proof of the Kuga-Satake-Deligne correspondence for these K3 surfaces and any K3 surfaces isogenous to them, and further, a proof of the Hodge conjecture for the squares of these surfaces. We conclude that the motives of these surfaces are Kimura-finite. Our analysis gives a birational equivalence between a moduli space of curves with additional data and the moduli space of these K3 surfaces with a specific elliptic fibration. This is joint work with Adam Logan and Owen Patashnick.

ELLEN KIRKMAN, Wake Forest University

Degree bounds for Hopf actions on Artin-Schelter regular algebras

In 1915 E. Noether proved that for a field \mathbb{k} of characteristic zero and a finite group G acting naturally on a polynomial ring $\mathbb{k}[x_1, \dots, x_n]$, the degrees of minimal generators of the subring of invariants are bounded above by the order of the group. In

2011, using Castelnuovo-Mumford regularity, P. Symonds proved that for a general field \mathbb{k} , an upper bound is $n(|G| - 1)$ when $n \geq 2$ and $|G| > 1$. Replacing $\mathbb{k}[x_1, \dots, x_n]$ by an Artin-Schelter regular algebra A and G by a semisimple Hopf algebra H , we prove analogues of results of Noether, Fogarty, Fleischmann, Derksen, Sidman, Chardin and Symonds on bounds on the degrees of generators of the subring of invariants and on the degrees of syzygies of modules over the invariant subring. We further explore Castelnuovo-Mumford regularity and related weighted sums of homological and internal degrees in complexes of graded A -modules for noncommutative algebras. This is joint work with Robert Won and James J. Zhang.

ALICIA LAMARCHE, University of Utah
Derived Categories, Arithmetic, and Rationality

When trying to apply the machinery of derived categories in an arithmetic setting, a natural question is the following: for a smooth projective variety X , to what extent can $\text{Db}(X)$ be used as an invariant to answer rationality questions? In particular, what properties of $\text{Db}(X)$ are implied by X being rational, stably rational, or having a rational point? On the other hand, is there a property of $\text{Db}(X)$ that implies that X is rational, stably rational, or has a rational point? In this talk, we will examine a family of arithmetic toric varieties for which a member is rational if and only if its bounded derived category of coherent sheaves admits a full étale exceptional collection. Additionally, we will discuss the behavior of the derived category under twisting by a torsor, which is joint work with Matthew Ballard, Alexander Duncan, and Patrick McFaddin.

MAX LIEBLICH, University of Washington
Filtered derived equivalence and birational Torelli theorems

I will report on joint work with Martin Olsson. We conjecture that two varieties admitting a filtered derived equivalence, in a sense that I will describe, must be birational. While we can prove various cases of this conjecture, we have not yet found a general mechanism that might yield a proof.

TONY PANTEV, University of Pennsylvania
Mirror symmetry, intersection of quadrics, and Hodge theory

I will discuss a construction of the homological mirror correspondence on algebraic integrable systems arising as moduli of flat bundles on curves. The focus will be on non-abelian Hodge theory as a tool for constructing objects in the Fukaya category. I will discuss specific example of the construction building automorphic sheaves on moduli space of bundles that are realized as intersections of quadrics. This is a joint work with Ron Donagi and Carlos Simpson.