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Improving estimates for discrete polynomial averages and related problems

For a polynomial P mapping the integers into the integers, define an averaging operator $A_N f(x) := \frac{1}{N}\sum_{k=1}^{N} f(x+P(k))$ acting on functions on the integers. We prove sufficient conditions for the ℓ^p-improving inequality

$$\|A_N f\|_{\ell^q(\mathbb{Z})} \lesssim_{P,p,q} N^{-d(\frac{1}{p}-\frac{1}{q})}\|f\|_{\ell^p(\mathbb{Z})}, \qquad N \in \mathbb{N},$$

where $1 \leq p \leq q \leq \infty$. For a range of quadratic polynomials, the inequalities established are sharp, up to the boundary of the allowed pairs of (p,q). For degree three and higher, the inequalities are close to being sharp. In the quadratic case, we appeal to discrete fractional integrals as studied by Stein and Wainger. In the higher degree case, we appeal to the Vinogradov Mean Value Theorem, established by Bourgain, Demeter, and Guth. We will also discuss some related problems for discrete averaging operators.