I will describe an approach to computing Hecke operators on the integral cuspidal cohomology of congruence subgroups of $SL_2(\mathcal{O}_d)$ over various rings of quadratic integers $\mathcal{O}_d$. The approach makes use of an explicit contracting homotopy on a classifying space for $SL_2(\mathcal{O}_d)$. The approach, which has been partially implemented, is also relevant for computations on congruence subgroups of $SL_m(\mathcal{Z})$, $m \geq 2$ (where it has been fully implemented for $m = 2$).