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The height of Mallow trees

Random binary search trees are obtained by recursively inserting the elements $\sigma(1), \sigma(2), \ldots, \sigma(n)$ of a uniformly random permutation σ of $[n] = \{1, \ldots, n\}$ into a binary search tree data structure. Devroye (1986) proved that the height of such trees is asymptotically of order $c^* \log n$, where $c^* = 4.311\ldots$ is the unique solution of $c \log((2e)/c) = 1$ with $c \geq 2$. Here, we study the structure of binary search trees $T_{n,q}$ built from Mallows permutations. A $\mathrm{Mallows}(q)$ permutation is a random permutation of $[n] = \{1, \ldots, n\}$ whose probability is proportional to $q^{\mathrm{Inv}(\sigma)}$, where $\mathrm{Inv}(\sigma) = \#\{i < j : \sigma(i) > \sigma(j)\}$. This model generalizes random binary search trees, since $\mathrm{Mallows}(q)$ permutations with $q = 1$ are uniformly distributed. The laws of $T_{n,q}$ and $T_{n,q^{-1}}$ are related by a simple symmetry (switching the roles of the left and right children), so it suffices to restrict our attention to $q \leq 1$.

We show that, for $q \in [0, 1]$, the height of $T_{n,q}$ is asymptotically $(1 + o(1))(c^* \log n + n(1 - q))$ in probability. This yields three regimes of behaviour for the height of $T_{n,q}$, depending on whether $n(1 - q)/\log n$ tends to zero, tends to infinity, or remains bounded away from zero and infinity. In particular, when $n(1 - q)/\log n$ tends to zero, the height of $T_{n,q}$ is asymptotically of order $c^* \log n$, like it is for random binary search trees. Finally, when $n(1 - q)/\log n$ tends to infinity, we prove stronger tail bounds and distributional limits for the height of $T_{n,q}$.