## Applications and Recent Developments in Discontinuous Dynamical Systems
## Applications et Avancées Récentes dans la Théorie des Systèmes Dynamiques Manifestant des Discontinuités

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**EVERALDO DE MELLO BONOTTO**, University of Sao Paulo

*Impulsive semidynamical systems*

Impulsive systems are used to describe the evolution of process whose continuous dynamics are interrupted by abrupt changes of state. In this talk, we present an overview of the theory of impulsive semidynamical systems.

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**ELENA BRAVERMAN**, University of Calgary

*Stabilization of cycles with impulse stochastic control*

We consider models of population dynamics and stabilize a prescribed cycle or an equilibrium of the difference equation using impulse stochastic control. Our technique, inspired by the Kolmogorov's Law of Large Numbers, activates a stabilizing effect of stochastic perturbation and allows for stabilization using a much wider range for the control parameter than would be possible in the absence of noise. Our main general result applies to both Prediction-Based and Target-Oriented Controls. This analysis is the first to make use of the stabilizing effects of noise for Prediction-Based Control; the stochastic version has been examined in the literature, but only the destabilizing effect of noise was demonstrated. A stochastic variant of Target-Oriented Control has never been considered, to the best of our knowledge, and we propose a specific form that uses a point equilibrium or one point on a cycle as a target. We demonstrate our results numerically on the logistic, Ricker and Maynard Smith models. This is joint work with C. Kelly (University College Cork) and A. Rodkina (University of the West Indies).

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**KEVIN CHURCH**, McGill

*Spectral theory for impulsive delay differential equations*

I will present some recent work on spectral theory for impulsive functional differential equations, with an emphasis on systems with discrete delays. For systems with periodic structure, this includes the associated Floquet theory. I will conclude with some applications to stability and bifurcation in mathematical models from ecology and infectious diseases.

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**GABRIEL DUCHESNE**, McGill University

*Rigorous computations of periodic solutions for the pulse-harvested Hutchinson equation*

I will present a rigorous numerical method to prove the existence of periodic solutions for the pulse-harvested Hutchinson equation. I will also briefly explain how we can extend this method to prove the existence of a global branch of periodic solutions.

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**MARCIA FEDERSON**, Universidade de São Paulo, Brazil

*An overview on stability results for impulsive and measure functional differential equations*

In this talk, we present results on stability, uniform stability and asymptotic stability for measure functional differential equations via generalized ODEs in the sense of J. Kurzweil. Impulsive problems are dealt with as a consequence of the main results.

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**XINZHI LIU**, University of Waterloo

*Impulsive Formation Control of Multi-Agent Systems*

As one of the most significant issues in the distributed coordination of multi-agent systems, formation control has received increased attention in recent years due to its wide applications in satellite formation flying, exploration, surveillance and rescue. The formation control problem aims to design suitable protocols such that a group of agents can reach a desired geometric structure from arbitrary initial positions. This talk discusses multi-group formation of multi-agent systems with multiple leaders. An impulsive protocol is proposed to take into consideration of intermittent communications among agents on a sequence of discrete times. It is shown, by employing results from graph theory and dynamical systems, that agents may be divided into multiple subgroups to follow different leaders while maintaining desired sub-formation configurations.

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**FRITHJOF LUTSCHER**, University of Ottawa
*Population dynamics of discrete breeders*

Many species are discrete (annual) breeders who, between reproductive events, consume resources and may die. Their resource often reproduces continuously or has short, overlapping generations. An accurate model for such life cycles needs to represent both, the discrete and the continuous processes in the community.

I will present a basic model for a single consumer and its resource in a two-season environment. I will give some basic properties of the model and explain how it differs from the purely continuous and the purely discrete analogues that have been studied for many decades. Then I will expand the model in two aspects: (i) I will consider coexistence mechanisms for many discrete-breeder consumers on a single limiting resource, and (ii) I will introduce spatial movement and present conditions for Turing pattern formation in such systems. This is joint work with Yunfeng Geng and Xiaoying Wang.

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**TYLER MEADOWS**, University of Idaho
*Self-cycling fermentation with a produced compound*

Self-cycling fermentation is a bioengineering process that is used in biofuel production and wastewater treatment. Due to some large differences in time-scales, the process can be modeled using a system of impulsive differential equations. Recently, engineers have been exploring different methods to trigger the discontinuous portion of the dynamics, called the decanting process. We examine a model of the self-cycling fermentation process in which the concentration of a produced compound is used to trigger the decanting process.

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**IAIN MOYLES**, York University
*A model of phosphorus recycling at the plant scale*

We present a model of phosphorus in soil that is taken up by a plant through its root systems. We consider the transformation of nutrient from labile forms in soil to above-ground biomass which is lost as leaf litter and is re-supplied to soil due to bacterial degradation. Since the plant roots are of a finite length, the removal term is discontinuous across the soil domain. Asymptotic analysis allows us to perform a model reduction that captures the phosphorus profile in various aspects of the soil. This has important implications in regional phosphorus management.

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**STACEY SMITH?**, The University of Ottawa
*Using non-smooth models to determine thresholds for microbial pest management*

Releasing infectious pests could successfully control and eventually maintain the number of pests below a threshold level. To address this from a mathematical point of view, two non-smooth microbial pest-management models with threshold policy are proposed and investigated in the present paper. First, we establish an impulsive model with state-dependent control to describe the cultural control strategies, including releasing infectious pests and spraying chemical pesticide. We examine the existence and stability of an order-1 periodic solution, the existence of order-k periodic solutions and chaotic phenomena of this model by analyzing the properties of the Poincaré map. Secondly, we establish and analyze a Filippov model. By examining the sliding dynamics, we investigate the global stability of both the pseudo-equilibria and regular equilibria. The findings suggest that we can choose appropriate threshold levels and control intensity to maintain the number of pests at or below the economic threshold. The modelling and control outcomes presented here extend the results for the system with impulsive interventions at fixed moments.

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**MARCO TOSATO**, York University
*Multi-cycle Periodic Solutions of a Differential Equation with Delay that Switches Periodically*

We analyse the dynamics of a scalar Delay Differential Equation with delay that periodically switches between two constant values. Such an equation arises naturally from structured vector populations involved in a range of vector-borne diseases spreading in a periodically varying environment. In particular, we show the example of a tick population model and how it can be described by this equation.

Then, we examine if and how the two different time lags and the switching time influence the existence and patterns of periodic solutions. We pay particular attention to the patterns involving multi-cycles within the prime period of the periodic solutions.

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**AILI WANG**, Baoji University of Arts and Sciences

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**GAIL WOLKOWICZ**, McMaster University
*Bifurcation analysis of an impulsive system describing Partial Nitritation and Anammox in a hybrid reactor*

Low-energy nitrogen removal under mainstream conditions is a technology that has received significant attention in recent years as the water industry drives towards long-term sustainability goals. Simultaneous partial nitritation-Anammox (PN/A) is one process that can provide substantial energy reduction and lower sludge yields. Mathematical modeling of the PN/A process offers engineers insights into the operating conditions necessary to maximize its potential. Laureni et al., Water Res. (2019) published a reduced mechanistic model of the process operated as a sequencing batch reactor, highlighting the effect of three key operating parameters on performance (Anammox biofilm activity, dissolved oxygen concentration, and fraction of solids wasted). Their analysis was limited to simulation with relatively few discrete parameter sets. We demonstrate using bifurcation theory applied to an impulsive system that the parameter space can be partitioned into regions in which the system converges to different fixed points that represent different outcomes: either the washout of nitrite oxidizing bacteria or their survival. Mapping process performance data onto these spaces allows engineers to target suitable operating regimes for specific objectives. We note that the nitrogen removal efficiency is maximized close to the curve of transcritical bifurcation points that separates the regions in parameter space where nitrite oxidizing bacteria washout from the region in which they survive. Further, control of solids washout and Anammox biofilm activity can also reduce oxygen requirements while maintaining an appropriate Hydraulic Retention Time. This approach is useful given the possibility for using such a methodology for models of increasing complexity.

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**KEXUE ZHANG**, University of Calgary
*A unified asymptotic stability result for time-delay systems with delayed impulses*

In this talk, we present a result of asymptotic stability of time-delay systems with delay-dependent impulses. This unified stability criterion can be applied to a variety of impulsive systems, such as systems with stabilizing continuous dynamics and destabilizing (or stabilizing) impulses, systems with destabilizing continuous evolution and stabilizing impulses, or systems with marginal stable continuous dynamics or marginal stable impulse effects. The unified stability criterion provides the (reverse) average dwell-time conditions on the impulse time sequences, and it is more general than the existing results in the sense that the derived stability guarantee does not require the uniform lower (and/or upper) bound of the impulse intervals. This is joint work with Elena Braverman (Calgary).