
MEGUMI HARADA, McMaster

Permutation bases for the cohomology rings of regular semisimple Hessenberg varieties.

Recent work of Shareshian and Wachs, Brosnan and Chow, and Guay-Paquet connects the well-known Stanley-Stembridge conjecture in combinatorics to the dot action of the symmetric group on the cohomology rings $H^*(\text{Hess}(S, h))$ of regular semisimple Hessenberg varieties. In particular, in order to prove the Stanley-Stembridge conjecture, it suffices to construct for any Hessenberg function h a permutation basis of $H^*(\text{Hess}(S, h))$ whose elements have stabilizers isomorphic to reflection subgroups. In this talk I will outline several recent results which contribute to this goal. Specifically, in some special cases, we give a new, purely combinatorial construction of classes in $H^*(\text{Hess}(S, h))$ which form permutation bases for subrepresentations in $H^*(\text{Hess}(S, h))$. Our techniques use the Goresky-Kottwitz-MacPherson theory in equivariant cohomology. Special cases of our construction have appeared in past work of Abe-Horiguchi-Masuda, Timothy Chow, and Cho-Hong-Lee. This is a report on joint work in progress with Martha Precup and Julianna Tymoczko.