TIMON GUTLEB, Imperial College London
Computing Equilibrium Measures with Power Law Kernels

Equilibrium measure problems naturally appear in the mathematical description of particle swarms in which particle behavior may be modeled via attractive and repulsive forces, for example ensemble movements in bird flocks, cellular scale organisms and classical particle interactions. Analytic solutions to equilibrium measure problems with power law kernels $K(x) = \frac{|x|^\alpha}{\alpha} - \frac{|x|^\beta}{\beta}$ exist for certain parameter choices but little is known about the behavior of solutions in high non-integer power cases, where discrete particle simulations predict interesting gap formation phenomena as the repulsive power increases. We introduce a banded sparse spectral method for such problems utilizing recurrence relationships in weighted ultraspherical polynomial bases. Numerical experiments agree with known analytic results as well as independent particle swarm simulations. Our method can be used to study solution behavior, uniqueness of solutions and the above-mentioned gap forming phenomenon.