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Computation of free boundary minimal surfaces via extremal Steklov eigenvalue problems

Recently Fraser and Schoen showed that the solution of a certain extremal Steklov eigenvalue problem on a compact surface with boundary can be used to generate a free boundary minimal surface, i.e., a surface contained in the ball that has (i) zero mean curvature and (ii) meets the boundary of the ball orthogonally (doi:10.1007/s00222-015-0604-x). We develop numerical methods that use this connection to realize free boundary minimal surfaces. Namely, on a compact surface, Σ, with genus γ and b boundary components, we maximize $\sigma_j(\Sigma, g)\ L(\partial\Sigma, g)$ over a class of smooth metrics, g, where $\sigma_j(\Sigma, g)$ is the j-th nonzero Steklov eigenvalue and $L(\partial\Sigma, g)$ is the length of $\partial\Sigma$. Our numerical method involves (i) using conformal uniformization of multiply connected domains to avoid explicit parameterization for the class of metrics, (ii) accurately solving a boundary-weighted Steklov eigenvalue problem in multi-connected domains, and (iii) developing gradient-based optimization methods for this non-smooth eigenvalue optimization problem. For genus $\gamma = 0$ and $b = 2, \ldots, 9, 12, 15, 20$ boundary components, we numerically solve the extremal Steklov problem for the first eigenvalue. The corresponding eigenfunctions generate a free boundary minimal surface, which we display in striking images. For higher eigenvalues, numerical evidence suggests that the maximizers are degenerate, but we compute local maximizers for the second and third eigenvalues with $b = 2$ boundary components and for the third and fifth eigenvalues with $b = 3$ boundary components. This is joint work with Braxton Osting (University of Utah) and Édouard Oudet (Université Grenoble Alpes).