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Planar domains with prescribed perimeter and large Steklov spectral gap must collapse to a point

In 2014, Gerasim Kokarev proved that the first nonzero Steklov eigenvalue of a compact surface Ω of genus 0 satisfies $\overline{\sigma}_1(\Omega) := \sigma_1(\Omega)|\partial\Omega| \leq 8\pi$. In a recent joint work with Jean Lagacé, we proved that this inequality is sharp by constructing a sequence of domains in the sphere $\mathbb{S}^2 \subset \mathbb{R}^3$ that saturates it. In an ongoing project with Mikhail Karpukhin and Jean Lagacé, we went further and proved the saturation of Kokarev inequality for planar domains: there exists a sequence $\Omega^\epsilon \subset \mathbb{R}^2$ such that $\overline{\sigma}_1(\Omega^\epsilon) \xrightarrow{\epsilon\to 0} 8\pi$. In this talk I will present a quantitative improvement of Kokarev's inequality, which sheds light on geometric and topological properties of such maximizing sequences for $\overline{\sigma}_1$. A particularly striking consequence is that any sequence $\Omega^\epsilon \subset \mathbb{R}^2$ with prescribed perimeter $|\Omega^\epsilon| = 1$ and $\sigma_1(\Omega^\epsilon) \xrightarrow{\epsilon\to 0} 8\pi$ accumulates at a point: Diameter$(\Omega^\epsilon) \xrightarrow{\epsilon\to 0} 0$. Another consequence is a quantitative lower bound on the number of connected components of the boundary $\partial\Omega$, which must grow to $+\infty$.