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Zeros of linear combinations of L-functions near the critical line

In this talk I will present a recent joint work with Yoonbok Lee, where we investigate the number of zeros of linear combinations of L-functions in the vicinity of the critical line. More precisely, we let $L_1, \ldots, L_J$ be distinct primitive L-functions belonging to a large class (which conjecturally contains all L-functions arising from automorphic representations on $\mathsf{GL}(n)$) and $b_1, \ldots, b_J$ be real numbers. Our main result is an asymptotic formula for the number of zeros of $F(s) = \sum_{j \leq J} b_j L_j(s)$ in the region $\mathrm{Re}(s) \geq 1/2 + 1/G(T)$ and $\mathrm{Im}(s) \in [T, 2T]$, uniformly in the range $\log \log T \leq G(T) \leq (\log T)^\nu$, where $\nu \asymp 1/J$. This establishes a generalization of a conjecture of Hejhal in this range.