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Arrangements of Consecutive Values of Real Multiplicative Functions

We will discuss the following problem: given a multiplicative function $f : \mathbb{N} \to \mathbb{R}$ and a k-tuple of "admissible", distinct non-negative integer shifts $a_1, \dots, a_k$, what is the probability that a given $n \in \mathbb{N}$ satisfies $f(n + a_1) \le \cdots \le f(n + a_k)$? Randomness heuristics suggest that such a pattern occur with probability $1/k!$ for a "generic" function f. Under certain assumptions on f we will give both conditional and unconditional results in this direction for a large collection of examples, in particular the Ramanujan τ function as well as sequences of Fourier coefficients of many non-CM, arithmetically normalized Hecke eigencusp forms with trivial nebentypus.