Optimal Transport and Applications
Transport Optimal et Applications
(Org: **Jun Kitagawa** (Michigan State) and/et **Abbas Momeni** (Carleton))

FARHAN ABEDIN, Michigan State University
Exponential Convergence of Parabolic Optimal Transport on Bounded Domains

I will speak about joint work with Jun Kitagawa on the asymptotic behavior of solutions to a parabolic version of the optimal mass transport problem. Our main result is an exponential rate of convergence for solutions of the evolution equation to the stationary solution of the optimal transport problem. The key ingredient we use is a global differential Harnack inequality for a special class of functions that solve the linearized problem. I will discuss the proof of this differential Harnack inequality in the case of domains with boundary, and show how it implies the desired exponential convergence result.

RENÉ CABRERA, University of Massachusetts Amherst
The Monge-Kantorovich Optimal Transportation of Mass Problem on Rectifiable Continuous Paths

The Monge-Kantorovich problem (MKP) is the study of transferring mass from an initial location to a final location in the most efficient way possible. Essentially, in the classical MKP we are a priori transferring mass along straight lines. But suppose that we now want to transfer mass along paths. In this talk, we present the MKP along paths and show that minimizers exist using a coercivity property. We consider maps $\Gamma(x,t) := (\gamma(x))(t)$ defined on the space of paths such that $\Gamma(x,0) = x$ and $\Gamma(x,1) = T(x)$, where T pushes forward μ_0 to μ_1—probability measures on $\overline{B}_R(0)$. Then for given transference plan on the space of paths, $\pi_\Gamma = \Gamma_\sharp \mu_0$, MKP takes the form $\int c(\gamma)\, d\pi(\gamma) = \int c(\Gamma)\, d\mu_0$. When the cost is $c(\gamma) := \int_0^1 \frac{1}{2}|\dot{\gamma}|^2$, perturbations and optimality conditions of Γ show π_Γ concentrates on constant speed geodesics. Then we untrivialize the problem on paths by adding congestion with interaction term: $\mathcal{E}(\pi) := \int c(\gamma) d\pi(\gamma) + \int\int\int_0^1\ u(|\gamma(t) - \sigma(t)|)\, dt d\pi(\gamma) d\pi(\sigma)$ between paths σ's, keeping γ fixed, with different endpoints. Here we require u to satisfying a Lipschitz condition. The methods that we used to prove existence of minimizers for MKP on paths apply equally well to this new formulation. Formally, the minimizers of $\mathcal{E}(\pi)$ are solutions of $-\partial_{tt}\Gamma(x,t) + \int\ u'(|\Gamma(x,t) - \Gamma(y,t)|) \frac{\Gamma(x,t)-\Gamma(y,t)}{|\Gamma(x,t)-\Gamma(y,t)|}\, d\mu_0(y) = 0$.

KATY CRAIG, University of California, Santa Barbara
A blob method for spatially inhomogeneous degenerate diffusion and applications to sampling and two layer neural networks.

Given a desired target distribution on Euclidean space and an initial guess of that distribution, composed of finitely many samples from Euclidean space, what is the best way to move the locations of the samples so that they more accurately represent the desired distribution? A classical solution to this problem is to allow the samples to evolve according to Langevin dynamics, the stochastic particle method for approximating solutions of the Fokker-Planck equation. In today's talk, I will introduce an alternative deterministic particle method for approximating solutions of a spatially inhomogeneous porous medium equation. This method corresponds exactly to the mean-field dynamics of training a two layer neural network for a radial basis function activation function. We prove that, as the number of samples increases and the variance of the radial basis function goes to zero, the particle method converges to a bounded entropy solution of the porous medium equation. As a consequence, we both obtain a novel method for sampling probability distributions as well as insight into the dynamics of training two layer neural networks in the mean field regime, including conditions on which the limiting energy is strongly convex. This is joint work with Karthik Elamvazhuthi (UCLA), Matt Haberland (Cal Poly), and Olga Turanova (Michigan State).

LUIGI DE PASCALE, University of Firenze
The relaxation of the Coulomb multi-marginal optimal transport cost and applications

Multi-marginal optimal transport costs are relevant for several applications. In particular the Coulomb repulsive cost play a role in certain energies in quantum mechanics. Some times, these energies don't allow to obtain compactness in the space of probability measures and then one needs to define, in a physically meaningful way, the transport cost for a sub-probability measure. After describing briefly the motivations, I will introduce the problem and two formula for the relaxed cost. If time allows I'll address the dual problem for the relaxed functional too. (From joints works with Guy Bouchitté, Giuseppe Buttazzo and Thierry Champion).

ALFRED GALICHON, New York University
Equilibrium transport with entropic regularization

In this talk, I will introduce and motivate the equilibrium transport problem, which is a generalization of the optimal transport problem, and I will focus on its entropic regularization. I will show the existence and uniqueness of a solution to a generalization of the Schrodinger-Bernstein system, and I will provide an algorithm to compute it. Based on joint work with Eugene Choo, Charles Liang, and Simon Weber.

SEONGHYEON JEONG, Michigan State University
Equivalence of the synthetic MTW conditions

I will present about the synthetic MTW conditions, Loerper's condition introduced by G. Loeper in 09 and QQconv introduced by N. Guillen and J. Kitagawa in 15. I will show that the two synthetic MTW conditions are equivalent when the regularity of the cost function is weaker than C^3.

YASH JHAVERI, Columbia University
On the (in)stability of the identity map in optimal transportation

In the optimal transport problem, it is well-known that the geometry of the target domain plays a crucial role in the regularity of the optimal transport. In the quadratic cost case, for instance, Caffarelli showed that having a convex target domain is essential in guaranteeing the optimal transport's continuity. In this talk, we shall explore how, quantitatively, important convexity is in producing continuous optimal transports.

YOUNG-HEON KIM, The University of British Columbia
Optimal transport for dendritic structures

Optimal transport gives an effective way to make geometric averages of different shapes, by giving a metric barycentre of a distribution over the space of probability measures. This metric barycentre is called the Wasserstein barycentre. We will discuss how this notion can be applied to studying dendritic structures, such as plant roots. Based on joint work with Brendan Pass (U. Alberta) and David Schneider (U. Saskatchewan).

TONGSEOK LIM, Purdue University
Geometry of interaction energy minimizers

An interaction energy is a quadratic function defined on a domain of probability measures. We study geometry and uniqueness of interaction energy minimizers on $\mathbb{R}^n$, $\mathbb{S}^n$, or $\mathbb{RP}^n$. This talk is based on a sequence of joint works with Robert J. McCann.

ROBERT MCCANN, University of Toronto
Inscribed radius bounds for lower Ricci bounded metric measure spaces with mean convex boundary

Consider an essentially nonbranching metric measure space with the measure contraction property of Ohta and Sturm. We prove a sharp upper bound on the inscribed radius of any subset whose boundary has a suitably signed lower bound on its generalized mean curvature. This provides a nonsmooth analog of results dating back to Kasue (1983) and subsequent authors. We prove a stability statement concerning such bounds and — in the Riemannian curvature-dimension (RCD) setting — characterize the cases of equality. This represents joint work with Annegret Burtscher, Christian Ketterer and Eric Woolgar.

ADRIAN TUDORASCU, West Virginia University
ON THE CONVEXITY CONDITION FOR THE SEMI-GEOSTROPHIC SYSTEM

We argue that conservative distributional solutions to the Semi-Geostrophic system in a rigid domain are in some well-defined sense critical points of a time-shifted energy functional involving measure-preserving rearrangements of the absolute density and momentum, which arise as one-parameter flow maps of continuously differentiable, compactly supported divergence free vector fields. We also show that the convexity requirement on the modified pressure potentials arises naturally if these critical points are local minimizers of said energy functional for any admissible vector field.

SHUANGJIAN ZHANG, CNRS, ENS Paris
Wasserstein Control of Mirror Langevin Monte Carlo

Discretized Langevin diffusions are efficient Monte Carlo methods for sampling from high dimensional target densities that are log-Lipschitz-smooth and (strongly) log-concave. In particular, the Euclidean Langevin Monte Carlo sampling algorithm has received much attention lately, leading to a detailed understanding of its non-asymptotic convergence properties and of the role that smoothness and log-concavity play in the convergence rate. Distributions that do not possess these regularity properties can be addressed by considering a Riemannian Langevin diffusion with a metric capturing the local geometry of the log-density. However, the Monte Carlo algorithms derived from discretizations of such Riemannian Langevin diffusions are notoriously difficult to analyze.

In this talk, we consider Langevin diffusions on a Hessian-type manifold and study a discretization that is closely related to the mirror-descent scheme. We establish for the first time a non-asymptotic upper-bound on the sampling error of the resulting Hessian Riemannian Langevin Monte Carlo algorithm. This bound is measured according to a Wasserstein distance induced by a Riemannian metric ground cost capturing the squared Hessian structure and closely related to a self-concordance-like condition. The upper-bound implies, for instance, that the iterates contract toward a Wasserstein ball around the target density whose radius is made explicit. Our theory recovers existing Euclidean results and can cope with a wide variety of Hessian metrics related to highly non-flat geometries. This talk represents joint work with Gabriel Peyré, Jalal Fadili, and Marcelo Pereyra.