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The Monge-Kantorovich Optimal Transportation of Mass Problem on Rectifiable Continuous Paths

The Monge-Kantorovich problem (MKP) is the study of transferring mass from an initial location to a final location in the most efficient way possible. Essentially, in the classical MKP we are a priori transferring mass along straight lines. But suppose that we now want to transfer mass along paths. In this talk, we present the MKP along paths and show that minimizers exist using a coercivity property. We consider maps $\Gamma(x,t) := (\gamma(x))(t)$ defined on the space of paths such that $\Gamma(x,0) = x$ and $\Gamma(x,1) = T(x)$, where T pushes forward μ_0 to μ_1—probability measures on $\overline{B}_R(0)$. Then for given transference plan on the space of paths, $\pi_\Gamma = \Gamma_\sharp \mu_0$, MKP takes the form $\int c(\gamma)\ d\pi(\gamma) = \int c(\Gamma)\ d\mu_0$. When the cost is $c(\gamma) := \int_0^1 \frac{1}{2}|\dot{\gamma}|^2$, perturbations and optimality conditions of Γ show π_Γ concentrates on constant speed geodesics. Then we untrivialize the problem on paths by adding congestion with interaction term: $\mathcal{E}(\pi) := \int c(\gamma) d\pi(\gamma) + \int \int \int_0^1\ u(|\gamma(t) - \sigma(t)|)\ dt d\pi(\gamma) d\pi(\sigma)$ between paths σ's, keeping γ fixed, with different endpoints. Here we require u to satisfying a Lipschitz condition. The methods that we used to prove existence of minimizers for MKP on paths apply equally well to this new formulation. Formally, the minimizers of $\mathcal{E}(\pi)$ are solutions of $-\partial_{tt}\Gamma(x,t) + \int\ u'(|\Gamma(x,t) - \Gamma(y,t)|) \frac{\Gamma(x,t)-\Gamma(y,t)}{|\Gamma(x,t)-\Gamma(y,t)|}\ d\mu_0(y) = 0$.