Algebraic Geometry of Integrable Systems
Géométrie Algébrique des Systèmes Intégrables

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ANA BALIBANU, Harvard University

Steinberg slices in quasi-Poisson varieties

We consider a multiplicative analogue of the universal centralizer of a semisimple group G — a family of centralizers parametrized by the regular conjugacy classes of the simply-connected cover of G. This multiplicative analogue has a natural symplectic structure and sits as a transversal in the quasi-Poisson double D(G). We show that D(G) extends to a smooth groupoid over the wonderful compactification of G, and we use this to construct a log-symplectic partial compactification of the multiplicative universal centralizer.

PETER CROOKS, Northeastern University

Hessenberg varieties and Poisson slices

Hessenberg varieties constitute a natural generalization of Grothendieck−Springer fibres, and their study lies at the interface of algebraic geometry, representation theory, and symplectic geometry. One defines Hessenberg varieties in the presence of Lie-theoretic data, which often include a complex semisimple Lie algebra $\mathfrak{g}$ with adjoint group G, the wonderful compactification $\overline{G}$ of G, and the log cotangent bundle $\mu : T^*\overline{G}(\log D) \longrightarrow \mathfrak{g} \oplus \mathfrak{g}$. The family of *standard Hessenberg varieties* is then a log symplectic Hamiltonian G-variety $\nu : \mathrm{Hess} \longrightarrow \mathfrak{g}$ bearing a close connection to the Kostant−Toda lattice. Balibanu has constructed a Poisson isomorphism

$$\mu^{-1}(\mathcal{S} \times \mathcal{S}) \cong \nu^{-1}(\mathcal{S}) \quad (*),$$

where $\mathcal{S} \subseteq \mathfrak{g}$ is a principal Slodowy slice. This allows one to embed generic fibres of ν into $\overline{G}$.

I will explain that $(*)$ extends to a G-equivariant Poisson bimeromorphism

$$\mu^{-1}(\mathfrak{g} \times \mathcal{S}) \cong \mathrm{Hess} \quad (**),$$

and that $(**)$ is an isomorphism if $\mathfrak{g} = \mathfrak{sl}_2$. This represents joint work with Markus Röser.

JACK DING, University of Toronto

Equivariant multiplicities of Schubert Varieties in the Based Loop Group

We compute equivariant multiplicities in the K-homology of Schubert varieties in the based loop group ΩG for the case of $G = SU(2)$. After taking a limit as the dimension of the Schubert varieties goes to ∞, we can interpret our results as an infinite-dimensional version of the well-known Atiyah-Bott-Lefschetz Formula. From this result we also derive an effective formula for computing characters of certain Demazure modules.

OLIVIA DUMITRESCU, University of North Carolina, Chapel Hill

Mirror curve of orbifold Hurwitz numbers

Abstract. Edge-contraction operations form an effective tool in various graph enumeration problems, such as counting Grothendieck's dessins d'enfants and simple and double Hurwitz numbers. Edge contraction operations were also used to define axioms of Topological Quantum Field Theory and Cohomological Field Theories.

These counting problems can be solved by a mechanism known as topological recursion. We investigate recursions of orbifold Hurwitz numbers, known as Cut-and-Join equations constructed solely from combinatorial data ie edge-contraction operations.

In particular we give an algebraic construction of the spectral curve of Hurwitz numbers obtained via recursion of the counting problem in genus 0 and one marked point.

This talk is based on joint work with Motohico Mulase.

IVA HALACHEVA, Northeastern University
Lagrangian correspondences in Schubert calculus

Given a reductive algebraic group G, it is a natural question to consider the inclusions of partial flag varieties H/Q into G/P and their pullbacks in equivariant cohomology, in terms of Schubert classes. We look at the case of the symplectic and usual Grassmannian, and describe a generalized construction involving Maulik-Okounkov classes and cotangent bundles of the Grassmannians, with Lagrangian correspondences playing a key role. This is joint work with Allen Knutson and Paul Zinn-Justin.

ELOISE HAMILTON, IMJ-PRG, University of Paris
Moduli spaces for unstable Higgs bundles of rank 2 and their geometry

The moduli space of semistable Higgs bundles is widely studied thanks to its rich geometric structure, in particular as it is an example of a Completely Integrable Hamiltonian System. In this talk we shift our focus from semistable to unstable Higgs bundles, guided by the questions of whether moduli spaces for unstable Higgs bundles can be constructed, and if so whether they admit a similarly rich geometric structure. We will start by considering the case of unstable (twisted) Higgs bundles of rank 2 on the projective line and by showing how moduli spaces can be constructed explicitly in this setting. We will then consider such Higgs bundles on a curve of arbitrary genus and explain how recent results in a generalisation of Geometric Invariant Theory (GIT), called Non-Reductive GIT, can be used to construct moduli spaces for them. We will finish by briefly describing initial steps towards understanding the geometry of these moduli spaces.

JACQUES HURTUBISE, McGill University
Moduli of bundles and degenerations of curves.

A good picture of the degeneration of moduli of bundles, corresponding to a degeneration of a smooth curve to a nodal curve would involve on one hand a good holomorphic family of moduli, and on the other, and in parallel, a compatible degeneration of the moduli of representations of the fundamental group of the surface, following Narasimhan and Seshadri. We present an approach to this problem. (joint with I. Biswas)

LISA JEFFREY, University of Toronto
The triple reduced product and Higgs bundles

We give an identification of the triple reduced product of three coadjoint orbits in $SU(3)$ with a space of Hitchin pairs $(\mathcal{E}, \Phi)$ over a genus 0 curve with three punctures, where the residues of Φ at the punctures are constrained to lie in fixed coadjoint orbits. In the language of Hitchin systems, we identify the moment map for a Hamiltonian circle action on this space of pairs. We make use of the results of Adams, Harnad and Hurtubise to find a description of this system.

DAVESH MAULIK, MIT
Cohomology of the moduli of Higgs bundles and the Hausel-Thaddeus conjecture

I will discuss some results on the structure of the cohomology of the moduli space of stable SL_n Higgs bundles on a curve. As a consequence, we obtain a new proof of the Hausel-Thaddeus conjecture, proven previously by Groechenig-Wyss-Ziegler via p-adic integration. If time allows, I will also mention connections with the P=W conjecture. This is joint work with Junliang Shen.

RUXANDRA MORARU, University of Waterloo
Moduli spaces of stable bundles on complex nilmanifolds

A *nilmanifold* is the quotient $N = \Gamma\backslash G$ of a connected, simply connected nilpotent Lie group G by a discrete, co-compact subgroup $\Gamma \subset G$. If N is equipped with a complex structure I induced by a left-invariant complex structure on G, then (N, I) is called a *complex nilmanifold*. Other than complex tori, examples of complex nilmanifolds are given by Kodaira surfaces and Iwasawa manifolds, to name a few. In this talk, I will present some interesting examples of moduli spaces of stable bundles on complex nilmanifolds.

ALEXEI OBLOMKOV, University of Massachusetts, Amherst
3D sigma models with defects and knot homology

Talk is based on the joint work with Lev Rozansky. In our work we construct a mathematical model for the gauged Kapustin-Rozansky-Saulina 3D sigma model with defects. The targets of the sigma model are the cotangent bundles to Lie algebras $\mathfrak{gl}_n$. The source of the sigma models is $\mathbb{R}^2 \times S^1$. In the case when the surface defect is of the form $C \times S^1 \subset \mathbb{R}^2 \times S^1$ the value of the partition function on the surface $\mathbb{R}^2 \times$ point is equal to the Khovanov-Rozansky homology of the knot that projects to C. Physics leads us to a geometric realisation of the Ocneanu-Jones trace in terms of sheaves on the Hilbert scheme of points on the plane. We use our constructions to explicitly compute the homology of torus knots. We also prove Poincare duality for the homology of knots, the duality that was conjectured by Dunfield-Gukov-Rasmussen in 2005.

BRENT PYM, McGill University
Beauville-Bogomolov-Weinstein splitting for Poisson varieties

The celebrated Beauville-Bogomolov and Weinstein decomposition theorems explain that certain geometries can be "split" as a product of smaller-dimensional geometries of the same type: the former is a global splitting for compact Kähler manifolds with trivial canonical class, while the latter is a local splitting for Poisson manifolds near a point on a symplectic leaf. I will describe a sort of fibre product of these results, governing the structure of complex projective Poisson manifolds. It shows, for instance, that after passing to an étale cover, a projective Poisson variety with a simply connected compact symplectic leaf splits as a product of said leaf and a projective Poisson variety containing a point where its Poisson bracket vanishes. The proof combines results from Hodge theory and holomorphic foliation theory with a recent notion of "subcalibations" for Poisson manifolds due to Frejlich and Mărcut in the differentiable setting. This talk is based on joint work with Stéphane Druel, Jorge Vitório Pereira and Frédéric Touzet.

JUNLIANG SHEN, MIT
Cohomological χ-independence for moduli of 1-dimensional sheaves and moduli of Higgs bundles

Let M_χ be either (a) the moduli space of 1-dimensional semistable sheaves F on a toric del Pezzo surface (e.g. $\mathbb{P}^2$) with $\chi(F) = \chi$, or (b) the moduli space of semistable Higgs bundles (E, θ) with respect to an effective divisor D on a curve of degree $\deg(D) > 2g - 2$ satisfying $\chi(E) = \chi$. Although the topology of the (possibly singular) variety M_χ relies heavily on χ, we show that the intersection cohomology of M_χ is indepedent of χ. This proves conjectures of Bousseau and Toda. In this talk, we will discuss particularly the role played by integrable systems in the χ-independence phenomenon. This is based on joint work in progress with Davesh Maulik.

SHIYU SHEN, University of Toronto
Topological mirror symmetry for parabolic Higgs bundles

I will present work on establishing the correspondence between the (appropriately defined) Hodge numbers of the moduli spaces of parabolic Higgs bundles for the structure groups $SL_n$and PGL_n, building on previous work of Groechenig-Wyss-Ziegler on the non-parabolic case. I will first describe the strategy used by Groechenig-Wyss-Ziegler, which combines p-adic integration with the generic duality between the Hitchin systems. Then I will talk about the new ingredients that come into play in the parabolic setting.