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On lifting invariant probability measures

I'll discuss the following question posed by Feliks Przytycki. Let X be a Polish space with a Borel probability measure μ. Let $T\colon X \to X$ be a pmp map. Let Y be a Polish space and $S\colon Y \to Y$ a continuous map. Suppose that $p\colon Y \to X$ is a Borel surjection such that $p \circ S = T \circ p$. Does μ lift to an S-invariant Borel probability measure ν on Y?

It turns out that if the sets $p^{-1}(x)$ are compact for all $x \in X$ then an S-invariant lift of μ exists. A similar result is true in a more general setting when an amenable semigroup acts on X and Y by pmp maps and continuous maps, respectively, and the actions commute with p. On the other hand, for non-amenable semigroups the result does not hold in general.