Nonlinear PDEs and kinetic problems
ÉDP non linéaires et problèmes cinétiques
(Org: **Slim Ibrahim** (Victoria) and/et **Weiran Sun** (SFU))

TAKAFUMI AKAHORI, Shizuoka university
Uniqueness of ground states for combined power-type nonlinear scalar field equations

We consider the uniqueness of ground states for combined power-type nonlinear scalar field equations with the Sobolev critical exponent and large frequency parameter. For five and higher dimensions, the uniqueness of the ground states had been proved. In this talk, I give a uniqueness result for three and four dimensions. This study is motivated and inspired by that by Coles and Gustafson (Publ.Res.Inst.Math.Sci.56 (2020), pp.647-699).

RICARDO ALONSO, Texas A&M Qatar
Brief Intro to Dissipative Particle Systems and the role of self-similarity

This talk is a brief introduction to Dissipative Particle Systems. The presentation evolves around 3 relevant examples: granular gases, alignment and annihilation processes. The notion of self-similarly is discussed and then connected to the analysis and simulation of such systems. Recent results and perspectives will be commented along the way.

YAKINE BAHRI, University of Victoria
Self-similar blow-up profiles for slightly supercritical nonlinear Schrödinger equations

We construct radially symmetric self-similar blow-up profiles for the mass supercritical nonlinear Schrödinger equation with nonlinear exponent close to the mass critical case and for any space dimension. These profiles bifurcate from the ground state solitary wave. In this talk, we present the argument which relies on the matched asymptotics method and we derive an exponentially smallness condition on the Sobolev critical exponent as conjectured by Sulem and Sulem in 1997.

This is a joint work with Yvan Martel and Pierre Raphaël.

GONG CHEN, Fields institute
Long-time dynamics of the sine-Gordon equation

I will discuss the soliton resolution and asymptotic stability for the sine-Gordon equation. It is known that the obstruction to the asymptotic stability for the sine-Gordon equation in the energy space is the existence of small breathers which is also closely related to the emergence of wobbling kinks. Our stability analysis gives a criterion for the weight which is sharp up to the endpoint so that the asymptotic stability holds. This is joint work with Jiaqi Liu and Bingying Lu.

I-KUN CHEN, National Taiwan University
A Revisit of the Velocity Averaging Lemma: On the Regularity of Stationary Boltzmann Equation in a bounded Convex Domain

We adopt the idea of velocity averaging lemma to establish regularity for stationary linearized Boltzmann equations in a bounded convex domain. Considering the incoming data, with three iterations, we establish regularity in fractional Sobolev space in space variable up to order 1-. This take is based on a joint work with Ping-Han Chuang, Chun-Hsiung Hsia, and Jhe-Kuan Su.

ZHENG CHEN, University of Massachusetts Dartmouth
Multiscale Convergence Properties for Spectral Approximation of a Model Kinetic Equation

We prove rigorous convergence properties for a semi-discrete, moment-based approximation of a model kinetic equation in one dimension. This approximation is equivalent to a standard spectral method in the velocity variable of the kinetic distribution and, as such, is accompanied by standard algebraic estimates of the form N^{-q}, where N is the number of modes and q depends on the regularity of the solution. However, in the multiscale setting, we show that the error estimate can be expressed in terms of the scaling parameter ϵ, which measures the ratio of the mean-free-path to the characteristic domain length. In particular, we show that the error in the spectral approximation is $\mathcal{O}(\epsilon^{N+1})$. More surprisingly, for isotropic initial conditions, the coefficients of the expansion satisfy super convergence properties. In particular, the error of the l^{th} coefficient of the expansion scales like $\mathcal{O}(\epsilon^{2N})$ when $l = 0$ and $\mathcal{O}(\epsilon^{2N+2-l})$ for all $1 \leq l \leq N$. This result is significant, because the low-order coefficients correspond to physically relevant quantities of the underlying system. All the above estimates involve constants depending on N, the time t, and the initial condition. We investigate specifically the dependence on N, in order to assess whether increasing N actually yields an additional factor of ϵ in the error. Numerical tests will also be presented to support the theoretical results.

YANXIA DENG, Sun Yat-sen University
Global existence and singularity of the Hill's type lunar problem

In a joint work with Slim Ibrahim, we used the idea of ground states in nonlinear dispersive equations (e.g. Klein-Gordon and Schrödinger equations) to characterize solutions in the N-body problem with strong force under some energy constraints. In this talk, I will explore this method to a restricted 3-body problem (Hill's type lunar problem), and talk about the dynamics of the solutions below, at, and (slightly) above the ground state energy threshold.

RAZVAN FETECAU, Simon Fraser University
Aggregation with intrinsic interactions on Riemannian manifolds

We consider a model for collective behaviour with intrinsic interactions on Riemannian manifolds. We establish the well-posedness of measure solutions, defined via optimal mass transport, on several specific manifolds (sphere, hypercylinder, rotation group SO(3)), and investigate the mean-field particle approximation. We study the long-time behaviour of solutions, where the primary goal is to establish sufficient conditions for a consensus state to form asymptotically. The analytical results are illustrated with numerical experiments that exhibit various asymptotic patterns.

HIROAKI KIKUCHI, Tsuda University
Existence of a ground state and blowup problem for a class of nonlinear Schrödinger equations

In this talk, we study the existence of the ground state and blowup problem for a class of nonlinear Schrödinger equations involving the mass and energy critical exponents. To show that a ground state exists, we solve a minimization problem related to the virial identity, so that we need to compare the minimization value to the best constant of the Gagliardo-Nirenberg inequality because our nonlinearities contain the mass critical nonlinearity. Once we obtain the ground state, we can introduce a subset $\mathcal{A}_{\omega,-}$ of $H^1(\mathbb{R}^d)$ for each $\omega > 0$ as in Berestycki and Cazenave (1981). Then, it turn out that any radial solution starting from $\mathcal{A}_{\omega,-}$ blows up in a finite time. This talk is based on a joint work with Minami Watanabe (Tsuda University).

KAI KOIKE, Kyoto University
Refined pointwise estimates for the solutions to a system of a 1D viscous compressible fluid and a moving point mass

The long-time behavior of a system of a one-dimensional barotropic viscous compressible fluid and a moving point mass is investigated. In a previous work, I showed that the velocity $V(t)$ of the point mass satisfies a power-law decay estimate $V(t) = O(t^{-3/2})$. This time, I give a necessary and sufficient condition for a corresponding lower bound $|V(t)| \geq C^{-1}(t+1)^{-3/2}$ ($t \gg 1$) to hold (preprint: https://arxiv.org/abs/2010.06578). This is proved as a corollary to refined pointwise estimates for the fluid variables.

QUYUAN LIN, Texas A&M University
The Inviscid Primitive Equations and the Effect of Rotation

Large scale dynamics of the oceans and the atmosphere is governed by the primitive equations (PEs). It is well-known that the three-dimensional viscous primitive equations are globally well-posed in Sobolev spaces. In this talk, I will discuss the ill-posedness in Sobolev spaces, the local well-posedness in the space of analytic functions, and the finite-time blowup of solutions to the three-dimensional inviscid PEs with rotation (Coriolis force). Eventually, I will also show, in the case of "well-prepared" analytic initial data, the regularizing effect of the Coriolis force by providing a lower bound for the life-span of the solutions which grows toward infinity with the rotation rate. The latter is achieved by a delicate analysis of a simple limit resonant system whose solution approximate the corresponding solution of the 3D inviscid PEs with the same initial data.

DAYTON PREISSL, University of Victoria
The Hot, Magnetized Relativistic Maxwell Vlasov System

Fusion energy is at the threshold of becoming one of the most green and sustainable energy sources in the world. This energy is creating by heating an ionized gas (plasma) to extreme temperatures in order to allow high energy particle collisions to occur. This leads to an exothermic fusion reaction releasing immense energy to be harvested. One major hurdle, is the plasma is highly pressurizes and must be contained within a reactor. A solution to this issue is applying a strong magnetic field which traps the particles from escaping radially outwards from the confinement chamber. Such a system can be modeled mathematically by the Hot, Magnetized, Relativistic Vlasov Maxwell (HMRVM) system. A small physically pertinent parameter ϵ, with $0 < \epsilon \ll 1$, related to the inverse of a gyrofrequency, governs the strength of a spatially inhomogeneous applied magnetic field given by the function $x \mapsto \epsilon^{-1}\mathbf{B}_e(x)$. Stationary (equilibrium) solutions to this system are well understood, but it is not clear how perturbations from equilibrium could lead to destabilization of the plasma (the plasma explodes releasing uncontrollable energy). It has been recently in shown that, in the case of *neutral*, *cold*, and *dilute* plasmas (like in the Earth's magnetosphere), smooth solutions corresponding to perturbations of equilibria exist on a uniform time interval $[0, T]$, with $0 < T$ independent of ϵ. In this talk we further extend these results to hot plasmas for well prepared initial data.

IKKEI SHIMIZU, Kyoto University
Local well-posedness for the Landau-Lifshitz equation with helicity term

We consider the initial value problem for the Landau-Lifshitz equation with helicity term (chiral interaction term), which arises from the Dzyaloshinskii-Moriya interaction. We show that it is locally well-posed in $\vec{k} + H^s$ for $s > 2$ with $\vec{k} = {}^t(0, 0, 1)$. The key idea is to reduce the problem to a system of semi-linear Schrödinger equations, called modified Schrödinger map equation. The problem here is that the helicity term appears as quadratic derivative nonlinearities, which is known to be difficult to treat as perturbation of the free evolution. To overcome that, we consider them as magnetic terms, then apply the energy method by introducing the differential operator associated with magnetic potentials.

TONG YANG, City University of Hong Kong
Some recent progress on the Boltzmann equation without angular cutoff

In this talk, after reviewing the work on global well-posedness of the Boltzmann equation without angular cutoff with algebraic decay tails, we will present a recent work on the global weighted L^∞-solutions to the Boltzmann equation without angular cutoff in the regime close to equilibrium. A De Giorgi type argument, well developed for diffusion equations, is crafted in this kinetic context with the help of the averaging lemma. More specifically, we use a strong averaging lemma to obtain suitable L^p estimates for level-set functions. These estimates are crucial for constructing an appropriate energy functional to carry out the De Giorgi argument. Then we extend local solutions to global by using the spectral gap of the linearized Boltzmann operator with the convergence to the equilibrium state obtained as a byproduct. This result fill in the gap of well-posedness theory for the Boltzmann equation without angular cutoff in the L^∞ framework. The talk is based on the joint works with Ricardo Alonso, Yoshinori Morimoto and Weiran Sun.

SHUGO YASUDA, University of Hyogo
Numerical analysis of the instability and aggregation in a kinetic transport equation with internal state

Collective motion of chemotactic bacteria, such as E. Coli, stems from, at individual level, continuous reorientations by runs and tumbles. It has been established that the length of run is decided by a stiff response to the external chemical cue via the intracellular signal transduction pathway. This study numerically investigates the self-organized aggregation of chemotactic bacteria based on a kinetic transport equation with internal state coupled with a reaction-diffusion equation of chemical cues. We put the focus on the effect of the adaptation time in the intracellular dynamics on the self-organized aggregation both at the macroscopic and microscopic levels. We found that the aggregation profile is highly affected by the adaptation time. Especially, we uncovered a non-monotonic behavior of the peak aggregation density with respect to the adaptation time. This indicates that there exists an optimal adaptation time to perform a strong aggregation behavior. Remarkably, this non-monotonic behavior is observed only at the kinetic level when the adaptation time is moderately large compared to the tumbling frequency, but cannot be described at the continuum level, i.e., the Keller-Segel model, which is obtained by the asymptotic analysis of the kinetic model. We also discover a plateau-like aggregation profile when the adaptation time is very large. We illustrate the formation of the plateau-type aggregation by a microscopic characterization.