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Multiscale Convergence Properties for Spectral Approximation of a Model Kinetic Equation

We prove rigorous convergence properties for a semi-discrete, moment-based approximation of a model kinetic equation in one dimension. This approximation is equivalent to a standard spectral method in the velocity variable of the kinetic distribution and, as such, is accompanied by standard algebraic estimates of the form N^{-q}, where N is the number of modes and q depends on the regularity of the solution. However, in the multiscale setting, we show that the error estimate can be expressed in terms of the scaling parameter ϵ, which measures the ratio of the mean-free-path to the characteristic domain length. In particular, we show that the error in the spectral approximation is $\mathcal{O}(\epsilon^{N+1})$. More surprisingly, for isotropic initial conditions, the coefficients of the expansion satisfy super convergence properties. In particular, the error of the l^{th} coefficient of the expansion scales like $\mathcal{O}(\epsilon^{2N})$ when $l = 0$ and $\mathcal{O}(\epsilon^{2N+2-l})$ for all $1 \leq l \leq N$. This result is significant, because the low-order coefficients correspond to physically relevant quantities of the underlying system. All the above estimates involve constants depending on N, the time t, and the initial condition. We investigate specifically the dependence on N, in order to assess whether increasing N actually yields an additional factor of ϵ in the error. Numerical tests will also be presented to support the theoretical results.