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From notational change to substantial discovery: Leibniz, Bernoulli, and the exponential notation for differentials

This paper revisits a famous episode of mathematical discovery in which it is often said that notations played a crucial role: Leibniz and Johann Bernoulli's discovery of an 'analogy between powers and differences', that is, of an analogy between the powers of a sum:

$$(x+y)^e = x^e + \frac{e}{1}x^{e-1}y^1 + \frac{e \cdot e-1}{1 \cdot 2}x^{e-2}y^2 + \frac{e \cdot e-1 \cdot e-2}{1.2.3}x^{e-3}y^3 \text{ etc.}$$

and the differentials of a product:

$$d^e(xy) = d^e x d^0 y + \frac{e}{1}d^{e-1}x \cdot d^1 y + \frac{e \cdot e-1}{1 \cdot 2}d^{e-2}x \cdot d^2 y + \frac{e \cdot e-1 \cdot e-2}{1 \cdot 2 \cdot 3}d^{e-3}x \cdot d^3 y \text{ etc.}$$

This discovery followed close on the heels of a notational innovation, namely Leibniz's introduction of an 'exponential' notation for differentials—for instance d^2x for ddx and d^3x for $dddx$, but also $d^{-1}x = \int x$—and the two developments are often presented as obviously related.

The goal of this talk is twofold: first, to clarify whether the notation indeed played a role by disentangling the specific contribution it may have made to the discovery from the motivations Leibniz may have had to introduce it in the first place; second, to contribute to a general investigation of how it is that notational choices—which may seem like mere abbreviations—can in fact shape the course of mathematical research. We shall see that, in this case, the notation did indeed make two significant contributions: it brought out a pattern in a particular formula which would have been less salient—harder to notice—otherwise; and it transformed what could be expressed in simple ways, thereby shaping further exploration.