Discrete Analysis
Analyse Discrète
(Org: **Tess Anderson** (Purdue), **Leo Goldmakher** (Williams) and/et **Brandon Hanson** (Georgia))

SAM CHOW, University of Warwick
Bohr sets in diophantine approximation

The correspondence between Bohr sets and generalised arithmetic progressions is a much-loved motif in additive combinatorics. We discuss the theory, and some applications, in the context of diophantine approximation.

MICHAEL CURRAN, University of Oxford
Khovanskii's Theorem and Effective Results on Sumset Structure

A remarkable theorem due to Khovanskii asserts that for any finite subset A of an abelian group, the cardinality of the h-fold sumset hA grows like a polynomial for all sufficiently large h. However, neither the polynomial nor what sufficiently large means are understood in general. We obtain an effective version of Khovanskii's theorem for any $A \subset \mathbb{Z}^d$ whose convex hull is a simplex; previously such results were only available for $d = 1$. Our approach also gives information about the structure of hA, answering a recent question posed by Granville and Shakan.

AYLA GAFNI, University of Mississippi
Asymptotics of Restricted Partition Functions

Given a set $\mathcal{A} \subset \mathbb{N}$, the restricted partition function $p_{\mathcal{A}}(n)$ counts the number of integer partitions of n with all parts in $\mathcal{A}$. In this talk, we will explore the features of the restricted partitions function $p_{\mathbb{P}_k}(n)$ where $\mathbb{P}_k$ is the set of k-th powers of primes. Powers of primes are both sparse and irregular, which makes $p_{\mathbb{P}_k}(n)$ quite an elusive function to understand. We will discuss some of the challenges involved in studying restricted partition functions and what is known in the case of primes, k-th powers, and k-th powers of primes.

LARRY GUTH, MIT
Incidence estimates for well spaced rectangles

We discuss estimating the overlap of thin rectangles in the plane in terms of how many rectangles clump together in fatter rectangles. This question can be seen as a generalization of the Szemeredi-Trotter theorem in incidence geometry, where straight lines are replaced by thin rectangles. Although the Szemeredi-Trotter theorem is sharp, there remain serious open problems involving these analogous questions for thin rectangles. We discuss a recent approach to the tube problem using Fourier analysis. This approach connects to decoupling and to the local smoothing problem for the wave equation.

MARINA ILIOPOULOU, University of Kent
A discrete Kakeya-type inequality

The Kakeya conjectures of harmonic analysis claim that congruent tubes that point in different directions rarely meet. In this talk we discuss the resolution of an analogous problem in a discrete setting (where the tubes are replaced by lines), and provide some structural information on quasi-extremal configurations. This is joint work with A. Carbery.

DOMINIQUE KEMP, Indiana University

OLEKSIY KLURMAN, University of Bristol
Zeros of Fekete polynomials

The study of the location of zeros of polynomials with coefficients constrained in different sets has a very rich history. The case of random polynomials has been extensively studied and the asymptotic number of real zeros has been computed in various cases (Gaussian, Bernoulli etc). The problem is subtler in a deterministic case. The goal of the talk is to briefly survey some developments in this subject and discuss recent progress for the family of Fekete polynomials. This is part of a joint work with Y. Lamzouri and M. Munsch.

ZANE LI, Indiana University, Bloomington
Connections between decoupling and efficient congruencing

We discuss a short proof of decoupling for the moment curve that is inspired from nested efficient congruencing. Connections between decoupling and efficient congruencing will also be highlighted. This talk is based off joint work with Shaoming Guo, Po-Lam Yung and Pavel Zorin-Kranich.

JOSE MADRID, University of California Los Angeles
Improving estimates for discrete polynomial averages and related problems

For a polynomial P mapping the integers into the integers, define an averaging operator $A_N f(x) := \frac{1}{N}\sum_{k=1}^{N} f(x+P(k))$ acting on functions on the integers. We prove sufficient conditions for the ℓ^p-improving inequality

$$\|A_N f\|_{\ell^q(\mathbb{Z})} \lesssim_{P,p,q} N^{-d(\frac{1}{p}-\frac{1}{q})}\|f\|_{\ell^p(\mathbb{Z})}, \qquad N \in \mathbb{N},$$

where $1 \leq p \leq q \leq \infty$. For a range of quadratic polynomials, the inequalities established are sharp, up to the boundary of the allowed pairs of (p,q). For degree three and higher, the inequalities are close to being sharp. In the quadratic case, we appeal to discrete fractional integrals as studied by Stein and Wainger. In the higher degree case, we appeal to the Vinogradov Mean Value Theorem, established by Bourgain, Demeter, and Guth. We will also discuss some related problems for discrete averaging operators.

AMITA MALIK, American Institute of Mathematics
Partitions into primes in arithmetic progression

In this talk, we discuss the number of ways to write a given integer as a sum of primes in an arithmetic progression. While the study of asymptotics for the number of ordinary partitions goes back to Hardy and Ramanujan, partitions into primes were recently re-visited by Vaughan. As a special case, we obtain an improvement in Vaughan's asymptotic formula for the number of partitions into primes.

FREDDIE MANNERS, UCSD
Some facts about very dense Sidon sets

A set S in an abelian group H is called a Sidon set if it has no non-trivial solutions to $x-y=z-w$ in S (i.e., with $\{x,w\} \neq \{y,z\}$). We say such a Sidon set is "very dense" if $|S| \geq (1-\varepsilon)|H|^{1/2}$, i.e., close to maximum possible size.

A variety of constructions for very dense Sidon sets exists in the additive combinatorics literature, and seemingly follow no shared pattern except that they all "come from algebra".

In this talk I will explain that they fit into a common framework: they all arise from letting H act on a finite projective plane by collineations.

These ideas essentially appeared a long time ago in the design theory literature, but seem less well known in additive combinatorics, so this talk functions as a sort of public service announcement. I will also discuss some related open questions.

Joint work with Sean Eberhard.

SARAH PELUSE, Institute for Advanced Study and Princeton University
Modular zeros in the character table of the symmetric group

In 2017, Miller conjectured, based on computational evidence, that for any fixed prime p the density of entries in the character table of S_n that are divisible by p goes to 1 as $n \to \infty$. I'll describe a proof of this conjecture, which is joint work with K. Soundararajan, along with proofs of some earlier results for small primes. I will also discuss the still open problem of determining the asymptotic density of zeros in the character table of S_n, where it is not clear from computational data what one should expect.

FELIPE RAMIREZ, Wesleyan University
Remarks about inhomogeneous pair correlations

A sequence a_n of natural numbers is said to have metric Poissonian pair correlations (MPPC) if for almost every real number α the associated sequence $\alpha a_n (\mathrm{mod} 1)$ on the circle has asymptotically Poissonian pair correlations. Informally speaking, this means that the points of the sequence clump together to the same extent that they would if they had been picked randomly. For example, the sequence of natural numbers does not have MPPC, while the sequence of square numbers does. Generally, if a sequence has too much additive structure, like the natural numbers, then it will not have MPPC. If it has very little additive structure, like the squares or the powers of 2, then it will have MPPC. But there is a zone in between "too much additive structure" and "very little additive structure" where the picture is not so clear, and there has been a lot of work devoted to finding an "additive energy threshold" separating sequences with MPPC from those without. I will survey this work, and I will discuss an associated inhomogeneous problem where the corresponding questions seem to be easier to answer.

FERNANDO SHAO, University of Kentucky
Gowers uniformity of primes in arithmetic progressions

A celebrated theorem of Green-Tao asserts that the set of primes is Gowers uniform, allowing them to count asymptotically the number of k-term arithmetic progressions in primes up to a threshold. In this talk I will discuss results of this type for primes restricted to arithmetic progressions. These can be viewed as generalizations of the classical Bombieri-Vinogradov theorem. I will also discuss a number of applications; for example, the set of primes p obeying $\{\sqrt{2}p^2\} < 0.1$ exhibit bounded gaps. This is joint work with Joni Teravainen.

ALED WALKER, Centre de Recherches Mathématiques
Effective results on the structure of sumsets

Given a finite set $A \subset \mathbb{Z}^d$ with convex hull $\mathrm{conv}(A)$, we have a trivial inclusion between the iterated sumset and the dilated convex hull, namely $NA \subset (N\,\mathrm{conv}(A)) \cap \mathbb{Z}^d$. But does equality ever hold? In fact there is an easily-described exceptional set $E_N(A)$ for which $E_N(A) \cap NA = \emptyset$, but one may nonetheless ask: does equality hold up to these exceptions?

Granville–Shakan recently showed that, if N is large enough, the answer to this question is yes, equality does hold. However, for all $d \geqslant 2$ their results gave only an ineffective lower bound on what 'large enough' should mean. In this talk we will describe two new pieces of work on this question: a new bound in the case $d = 1$, which is tight for several infinite families of sets A, and the first effective bounds for arbitrary A when $d \geqslant 2$. These results are joint work with Granville and with Granville–Shakan respectively. If time permits, we will describe the connections between this work and Khovanskii's theorem (that the size of NA is a polynomial in N, for large enough N).

HONG WANG, Institute for Advanced Study
Small cap decouplings

We will discuss some incidence estimates for tubes (and planks) using basic Fourier analysis, based on the joint work with Guth and Solomon. Then we show how these incidence estimates are used to prove decoupling inequalities. This is joint work with Ciprian Demeter and Larry Guth.

RUXIANG ZHANG, Institute for Advanced Study
Local smoothing for the wave equation in 2+1 dimensions

Sogge's local smoothing conjecture for the wave equation predicts that the local L^p space-time estimate gains a fractional derivative of order almost $1/p$ compared to the fixed time L^p estimates, when $p > 2n/(n-1)$. Jointly with Larry Guth and Hong Wang, we recently proved the conjecture in $\mathbb{R}^{2+1}$. I will talk about a sharp square function estimate we proved which implies the local smoothing conjecture in dimensions $2+1$. A key ingredient in the proof is an incidence type theorem.