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Homotopy theories of diagrams

A pro-object in simplicial sets is a diagram $X : I \to s\mathbf{Set}$ with I right filtered. The pro-object X represents a functor $\hom(X,\ )$ with

$$\hom(X, Z) = \varinjlim_{i \in I} \hom(X(i), Z),\ \text{for } Z \in s\mathbf{Set}.$$

A map $\phi : X \to Y$ of pro-objects is a map $\hom(Y,\ ) \to \hom(X,\ )$. There is a model structure (Edwards-Hastings) on this category for which a map ϕ is a weak equivalence if the map

$$\varinjlim_{j}\ \mathbf{hom}(Y(j), Z) \to \varinjlim_{i}\ \mathbf{hom}(X(i), Z)$$

(filtered colimits of function complexes) is a weak equivalence for all fibrant Z.

This talk describes a potential generalization of this structure to all small diagrams of simplicial sets, in a "pro-category" that is a Grothendieck construction: the objects are small diagrams $X : I \to s\mathbf{Set}$, and a morphism $\phi : X \to Y$ is a pair (α, f) consisting of a functor $\alpha : J \to I$ and a map of J-diagrams $f : X \cdot \alpha \to Y$, where $Y : J \to s\mathbf{Set}$ is another small diagram.