## STUDC Research Session

**KYLE BRYENTON**, University of Prince Edward Island
*Exactly Solvable Anharmonic Potentials with Variable Bumps and Depths*

A new approach based on Darboux Transformations is introduced to generate new classes of solvable anharmonic potentials with a variable number of bumps and depths. By introducing the concept of a transformation key, we present a method of controlling the number of bumps and their depths in these potentials. Although this method was applied to the one-dimensional generalized harmonic oscillator potential, it can be easily adapted to generate exactly-solvable potentials using other known quantum potentials.

**ADILBEK KAIRZHAN**, McMaster University
*Orbital instability of standing waves for NLS equation on star graphs*

We consider the nonlinear Schrodinger (NLS) equation with positive power nonlinearity on a star graph $\Gamma$ ($N$ half-lines glued at the common vertex) with a $\delta$ interaction at the vertex. The strength of the interaction is defined by a fixed real value $\alpha$. In the recent works of Adami et al., it was shown that for $\alpha < 0$ the NLS equation on $\Gamma$ admits the unique orbitally stable symmetric standing wave, while all other standing waves are asymmetric and were conjectured to be unstable. In this talk, we present the stability analysis of the asymmetric waves for $\alpha < 0$. By extending the Sturm theory to Schrodinger operators on the star graph, we give the explicit count of the Morse and degeneracy indices for each standing wave. Based on the count, we prove the orbital instability of the asymmetric standing waves.

**RAMSHA KHAN**, McMaster University
*A Variational Data Assimilation Scheme for Prediction of Ocean Bathymetry from Surface Waves*

Accurate mapping of ocean floor topography is a multi-faceted process, needed for safe and efficient navigation on shipping routes as well as for predicting tsunami waves. Currently available bathymetry data does not always provide a resolution high enough to capture the dynamics of such nonlinear waves accurately. However, collection of accurate mapping data is difficult, costly, and often a dangerous affair. As an alternative, theoretical approaches use propagation of free surface waves to extrapolate the shape of the bathymetry. In this study we implement a variational data assimilation scheme on the shallow water equations to improve estimates of the bathymetry, using observations of surface wave height to optimise our predictions. We show that a necessary condition is that the amplitude of the initial condition wave be much smaller relative to the amplitude of the bathymetry. If our objective is to use this reconstructed bathymetry to accurately predict tsunami waves, we observe that a relatively higher error in the optimal reconstruction of the bathymetry still yields a highly accurate prediction of the surface wave, suggesting low sensitivity of surface waves to higher frequencies in the bathymetry. We also analytically derive the sensitivity of our optimisation to perturbations in the observations. These conclusions are based on numerical experiments for both periodic and non-periodic bathymetry, and with different observation operators. These results can potentially have a significant impact in the real world, where computational cost can be minimised through a priori knowledge of sufficient error tolerances needed for accurate tsunami prediction.

**PRITPAL MATHARU**, McMaster University
*Optimal Closures in a Simple Model for Turbulent Flows*

Turbulent flows at high Reynolds numbers continue to challenge both scientists studying their fundamental properties and engineers interested in diverse technical applications involving fluid mechanics. In particular, accurate and efficient numerical simulation of turbulent flows will remain an open problem in computational science for the foreseeable future. As a result, one must rely on solving various simplifications and obtain approximate solutions of the flow problem. One such approach which has gained widespread popularity in engineering practice relies on the so-called Large-Eddy Simulation (LES). In this talk, I will introduce a computational framework for determining optimal closures of the eddy-viscosity type for LES of a broad class of PDE models, such as the Navier-Stokes equation. The proposed framework is thoroughly tested on a model problem involving the LES of the 1D Kuramoto-Sivashinsky equation, where the optimal closure relations are obtained as generalizations of the standard Smagorinsky model. Since this leads to a PDE optimization problem with a nonstandard structure, the solution is obtained computationally with a flexible and efficient gradient approach relying on a combination of modified adjoint-based analysis and Sobolev gradients, which will be discussed along with some results.

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**PIERRE-OLIVIER PARISÉ**, Université Laval

*On rank-one unitary perturbations of the shift operator over a de Branges-Rovnyak space*

Due to a celebrating Theorem of Beurling and Lax, the closed subspaces $K_\theta$, called the model spaces and parametrized by some function $\theta$, of the Hardy space $H^2$ characterize the closed invariant subspaces of the backward shift operator $S^*$ on the Hardy space $H^2$. This means that any closed subspace $V$ of $H^2$ such that $S^*V \subset V$ must be a model space $K_\theta$, that is $V = K_\theta$.

In connection with this result, in 1972, Clark studied the invariant subspaces of the restricted shift $S_\theta := P_{K_\theta}\, S|_{K_\theta}$ defined on the model space $K_\theta$ where $P_{K_\theta}$ is the orthogonal projection on $K_\theta$. One of his interests was to characterize the rank-one unitary perturbations of the restricted shift operator $S_\theta$. He showed that such operators must be of the form $U_\alpha$ where $U_\alpha f = S_\theta f + \langle \alpha f, S^*\theta \rangle\, 1$ where $\langle \cdot, \cdot \rangle$ denotes the inner product in $H^2$ and $|\alpha| = 1$.

The goal of the talk is to generalize Clark's result to the de Branges-Rovnyak spaces in the case of extreme points $b$, a generalisation of the model space $K_\theta$. More specifically, I will introduce the de Branges-Rovnyak spaces and show how the shift operator may be defined on this space. Then, I will show that Clark's result still hold in this framework.

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**SHEN QUANLI**, University of Lethbridge

*The fourth moment of quadratic Dirichlet L-functions*

In this talk, we will investigate the fourth moment of quadratic Dirichlet $L$-functions. Under the generalized Riemann hypothesis, we showed an asymptotic formula for the fourth moment. Unconditionally, we established a precise lower bound. This work is mainly based on the Soundararajan and Young's paper in 2010 on the second moment of quadratic twists of modular $L$-functions.

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**TESSA REIMER**, University of Manitoba

*Properties of Symbolic Powers of Ideals*

Symbolic powers of ideals in the polynomial ring have appeared in many significant problems in commutative algebra, algebraic geometry, and other branches of mathematics. However, their properties have proven challenging to study, even for square-free monomial ideals. In this talk, we will discuss a splitting for symbolic powers of edge ideals of graphs. This splitting can be used to explore the graded Betti numbers of the symbolic power which in turn provides insight into relationships between generators and syzygies. We will close with a discussion of bounds for Betti numbers of parallelizations of graphs