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Symmetry multi-reduction method for partial differential equations with conservation laws

For partial differential equations (PDEs) that have $n \geq 2$ independent variables and a symmetry algebra of dimension at least $n - 1$, an explicit algorithmic method is presented for finding all symmetry-invariant conservation laws that will reduce to first integrals for the ordinary differential equation (ODE) describing symmetry-invariant solutions of the PDE. This significantly generalizes the double reduction method known in the literature. Moreover, the condition of symmetry-invariance of a conservation law is formulated in an improved way by using multipliers, thereby allowing symmetry-invariant conservation laws to be obtained directly, without the need to first find conservation laws and then check their invariance. This cuts down considerably the number and complexity of computational steps involved in the reduction method. If the space of symmetry-invariant conservation laws has dimension $m \geq 1$, then the method yields m first integrals along with a check of which ones are non-trivial via their multipliers. Several examples of interesting symmetry reductions are considered: travelling waves and similarity solutions in $1 + 1$ dimensions; line travelling waves, line similarity solutions, and similarity travelling waves in $2 + 1$ dimensions; rotationally symmetric similarity solutions in $n + 1$ dimensions.