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Average states of quantum walks

A state of a finite dimensional quantum system can be specified by a *density matrix*, a positive semidefinite matrix with trace 1. The evolution of the system (in the simplest case) is determined by unitary matrices

$$U(t) := \exp(itA), \qquad (t \geq 0).$$

Here A is called the *Hamiltonian* of the system and must be Hermitian. If the initial state of the system is D, then its state at time t will be $U(t)DU(-t)$, which we denote by $D(t)$. For us, the Hamiltonian will be the adjacency matrix of a graph, in which case our quantum system is a *continuous quantum walk*. Continuous quantum walks are sometimes said to be analogs of classical continuous random walks, but the analogy is weak. In particular although classical random walks converge to a steady state under mild conditions, quantum walks do not.

However we can define the average state of a quantum walk with initial state D to be

$$\hat{D} := \lim_{T \to \infty} \frac{1}{T} \int_0^T D(t)\, dt$$

My talk will discuss some of the properties of these average states, and some of their uses.