Geometric Quantization: Old and New
Geometric Quantization
(Org: **Mark Hamilton** (Mount Alison University) and/et **Yael Karshon** (University of Toronto, Mississauga))

TATYANA BARRON, University of Western Ontario
The twistor space of $\mathbb{R}^{4n}$ and Berezin-Toeplitz quantization.

On the hyperkähler manifold $\mathbb{R}^{4n}$ there is a family of induced complex structures parametrized by S^2 and a corresponding family of Berezin-Toeplitz quantizations. We use the twistor space of $\mathbb{R}^{4n}$ to construct a quantization, as a replacement to the family of quantizations above.

RICHARD CUSHMAN, University of Calgary
Bohr-Sommerfeld-Heisenberg quantization of the mathematical pendulum

For a completely integrable Hamiltonian system, Bohr-Sommerfeld quantization of the action variables gives rise to a space of quantum states and a complete set of commuting observables acting of this space of states. Bohr-Sommerfeld theory does not provide operators of transition between the eigenstates of operators corresponding to the actions. These transitions are accounted for by shifting operators. Their existence for the mathematical pendulum will be discussed in the talk. The commutation relations satisfied by the shifting operators are the same as the commutation relations satisfied by formal quantization of the exponential function of i times theta, where theta is an angle in the action angle coordinates for the integrable system. If theta were a single-valued function, then its Hamiltonian vector field would generate a local group of local symplectomorphisms of the phase space preserving the Bohr-Sommerfeld polarization by level sets of the Hamiltonian. This flow would lift to a local group of local quantomorphisms of the quantum line bundle. Since the angle theta is a multi-valued function, this local group of quantomorphisms is not well defined. However, the shifting operators, given by evaluating the lifted flow of quantomorphisms at $t = h$ is well defined and corresponds to the operators of multiplication by the exponential function of i times theta. The existence of shifting operators answers Heisenberg's criticism of Bohr-Sommerfeld theory.

HAJIME FUJITA, Japan Women's University
A K-homology cycle via perturbation by Dirac operators along orbits

Recently Loizides-Song developed a geometric quantization of Hamiltonian loop group space in terms of KK-theory. In their work a K-homology cycle constructed from a Dirac operator on a non-compact complete manifold has crucial role. They constructed such a cycle based on a C^*-algebraic condition for vector bundles. On the other hand, as a joint work with M.Furuta and T.Yoshida, the speaker developed an index theory based on perturbation by Dirac operators along orbits of torus action, and gave applications to localization phenomena to lattice points in geometric quantization. In this talk we will report ongoing work to construct a Loizides-Song type K-homology cycle via perturbation by Dirac operator along orbits.

YUCONG JIANG, University of Toronto
Pairing maps in geometric quantization

I will review Blattner's half form pairing of different polarizations and mention its generalization in metaplectic-c quantization.

PANEL DISCUSSION,

JEDRZEJ SNIATYCKI, University of Calgary
Bohr-Sommerfeld Quantum System

We have shown that geometric quantization applied to a completely integrable system with a Bohr-Sommerfeld polarization leads to complete quantum theory [R.Cushman and J. Sniatycki, ["Shifting Operators in Geometric Quantization", arXiv: 1808.04002v2 [math.SG]]. The original Bohr-Sommerfeld theory constructs the space of states and gives spectra of operators corresponding to functions constant along the polarization. Geometric quantization constructs a family of intrinsic shifting operators acting transitively on the space of states. Our results refute Heisenberg's criticism that in the Bohr-Sommerfeld theory there are not enough operators to describe transitions between states. In fact, these operators exist, but it took quite a while to find them.

ALEJANDRO URIBE, University of Michigan
Squeezed coherent states in complex polarizations

A quantized Kähler manifold has a natural family of coherent states, namely, those given by freezing the first entry of the Bergmann projection. Such states however do not preserve their shape under a quantum evolution. I will describe how to construct more general coherent states in a couple of different geometric ways, and state a theorem about their quantum evolution in the semi-classical limit.

JENNIFER VAUGHAN, University of Manitoba
A Brief Tour of Metaplectic-c Prequantization

A metaplectic-c prequantization of a symplectic manifold is defined. Its construction is compared with Kostant-Souriau prequantization with half-form correction. Some results involving metaplectic-c prequantizations are surveyed, including calculations of quantized energy levels and a metaplectic-c equivariant version of the Delzant construction of toric manifolds.

JONATHAN WEITSMAN, Northeastern University
Towards a polarization-free quantization

We show how a variant of geometric quantization which is free of a choice of polarization may be defined, and also possible problems with this method.

TAKAHIKO YOSHIDA, Meiji University
Adiabatic limits, Theta functions, and Geometric quantization

In the geometric quantization of toric manifolds, Baier-Florentino-Mourão-Nunes have given a one-parameter family of complex structures such that the associated Kähler polarizations converge to the real polarization determined by the moment map. One of the generalizations of the Kähler quantization to possibly non-Kähler symplectic manifolds is the Spin^c quantization. In this talk, for a non-singular Lagrangian torus fibration on a compact, complete base with prequantum line bundle and a compatible almost complex structure invariant along the fiber, we show that the Spin^c quantization converges to the real quantization by the adiabatic(-type) limit. This talk is based on arXiv:1904.04076.

FRANCOIS ZIEGLER, Georgia Southern University
Quantization, after Souriau

"Quantization" attaches representations to (prequantized) coadjoint orbits. It's also a rather unprincipled, motley cookbook of answers to questions which are not always very clearly spelled out. Ever dissatisfied with this situation, J.-M. Souriau spent his last few papers in an effort to clarify what "attached" should mean. Interestingly, the notion he proposed relaxes such requirements as uniqueness, irreducibility on transitive subgroups, and even continuity. In this talk, I will explain how it 1°)

eschews “no-go theorems”; 2°) allows genuine eigenstates belonging to lagrangian submanifolds, as advocated by A. Weinstein; but 3°) is likely not quite restrictive enough.