GUIXIANG HONG, Wuhan University
Pointwise convergence of noncommutative Fourier series

This paper is devoted to the study of convergence of Fourier series for nonabelian groups and quantum groups. It is well-known that a number of approximation properties of groups can be interpreted as some summation methods and mean convergence of associated noncommutative Fourier series. Based on this framework, this work studies the refined counterpart of pointwise convergence of these Fourier series. We establish a general criterion of maximal inequalities for approximative identities of noncommutative Fourier multipliers. As a result we prove that for any countable discrete amenable group, there exists a sequence of finitely supported positive definite functions tending to 1 pointwise, so that the associated Fourier multipliers on noncommutative L_p-spaces satisfy the pointwise convergence for all $1 < p < \infty$. In a similar fashion, we also obtain results for a large subclass of groups (as well as discrete quantum groups) with the Haagerup property and weak amenability. We also consider the analogues of Fejer means and Bochner-Riesz means in the noncommutative setting. Our results in particular apply to the almost everywhere convergence of Fourier series of Lp-functions on non-abelian compact groups. On the other hand, we obtain as a byproduct the dimension free bounds of noncommutative Hardy-Littlewood maximal inequalities associated with convex bodies. As an ingredient, our proof also provides a refined version of Junge-Le Merdy-Xu's square function estimates $H_p(\mathcal{M}) \simeq L_p(\mathcal{M})$ when $p \to 1$.