Fractal Geometry, Analysis, and Applications
Gémoétrie fractale, analyse et applications
(Org: **Herb Kunze** (Guelph), **Franklin Mendivil** (Acadia) and/et **József Vass** (CAIS))

BERTRAND DUPLANTIER, Institute for Theoretical Physics, Paris-Saclay University
Integral Means Spectrum for Schramm-Loewner Evolution

I will present recent results on the integral means spectrum for Schramm-Loewner Evolution (SLE). This will include the complex generalized spectrum of whole-plane SLE, as well as the complex spectrum of whole-plane SLE as driven by Brownian motion with drift.

Based on joint works with D. Beliaev, Y. Han, H. Ho, B. Le, C. Nguyen, M. Zinsmeister.

BERNAT ESPIGULÉ, Universitat de Barcelona
Complex Trees: Structural Stability of Connected Self-similar Sets

The theory of complex trees is introduced as a new approach to study a broad class of self-similar sets which includes Cantor sets, Koch curves, Lévy C curves, Sierpinski gaskets, Rauzy fractals, plane-filling curves, and fractal dendrites. We note a fundamental dichotomy for n-ary complex trees that allows us to study topological changes in regions where one-parameter families of connected self-similar sets are defined. Moreover, we show how to obtain these families from systems of equations encoded by tip-to-tip equivalence relations. The parameter space maps that we introduce to study these families of connected self-similar sets are new. And for $T_A(z) := T\{z, \frac{1}{2}, \frac{1}{4z}\}$ we show that the boundary surrounding structurally stable trees is piecewise smooth.

KEVIN HARE, University of Waterloo
Entropy of Self-similar measures

It is known that a self-similar measure is either purely singular or absolutely continuous. Despite this, for most measures we cannot say which case we are in. One technique that has proved promising is the study of the Garsia Entropy of the measure. In this talk I will discuss the history, properties and recent results for self-similar measures and Garsia Entropy.

JUN KIGAMI, Kyoto University
Ahlfors regular conformal dimension of metric spaces and parabolic index of infinite graphs

In this talk, I am going to explain an analytic characterization of Ahlfors regular conformal dimension. Through this characterization, one can see a connection between Ahlfors regular conformal dimension of a metric space and the parabolic index of the infinite graph associated with a "brow-up" of the metric space.

HERB KUNZE, University of Guelph
Solving Inverse Problems using A Multiple Criteria Model with Collage Distance, Entropy, and Sparsity

In recent years, the Collage Theorem, a central result in fractal imaging, and some similarly-motivated results have been used to solve inverse problems in differential equations, integral equations, and other areas. In this talk, we extend the previous method for solving inverse problems for steady-state equations using the Generalized Collage Theorem by searching for an approximation that not only minimizes the collage error but also maximizes the entropy and minimize the sparsity. In this extended formulation, the parameter estimation minimization problem can be understood as a multiple criteria problem, with three different and conicting criteria: The generalized collage error, the entropy associated with the unknown parameters, and the sparsity of the set of unknown parameters. We implement a scalarization technique to reduce the multiple criteria program to a single criterion one, by combining all objective functions with different trade-off weights. Numerical examples confirm that the collage method produces good, but sub-optimal, results. A relatively low-weighted entropy term allows for better approximations, while the sparsity term decreases the complexity of the solution in terms of the number of elements in the basis.

MICHEL L. LAPIDUS, University of California, Riverside
An Introduction to Complex Fractal Dimensions

We will give some sample results from the new higher-dimensional theory of complex fractal dimensions developed jointly with Goran Radunovic and Darko Zubrinic in the 700-page research monograph (joint with these same co-authors), "Fractal Zeta Functions and Fractal Drums: Higher Dimensional Theory of Complex Dimensions" [2], published by Springer in 2017 in the Springer Monographs in Mathematics series. We will also explain its connections with the earlier one-dimensional theory of complex dimensions developed, in particular, in the research monograph (by the speaker and Machiel van Frankenhuijsen) entitled "Fractal Geometry, Complex Dimensions and Zeta Functions: Geometry and Spectra of Fractal Strings" [1] (Springer Monographs in Mathematics, Springer, New York, 2013; 2nd rev. and enl. edn.).

If time permits, we will discuss and extend to any dimension the general definition of fractality proposed by the author (and M-vF) in [1], as the presence of nonreal complex dimensions. Finally, we may also discuss fractal tube formulas which enable us to express the intrinsic oscillations of fractal objects in terms of the underlying complex dimensions and the residues of the associated fractal zeta functions. Intuitively, the real parts of the complex dimensions correspond to the amplitudes of the associated "geometric waves", while their imaginary parts correspond to the frequencies of those waves.

FARZANEH NIKBAKHTSARVESTANI, Department of Mathematics, University of Manitoba, Winnipeg, MB, Canada
On the existence of a solution for a multi- singular integro-differential equation with integral boundary conditions

In this paper, we investigate the existence of a solution for the pointwise defined, multi-singular fractional differential equation

$$D^{\alpha}x(t) = f(t, x(t), x'(t), D^{\beta}x(t), \int_0^t g(\xi)x(\xi)d\xi),$$

with some integral boundary conditions.

JÓZSEF VASS, CAIS
The Inverse Problem of Fractal Potentials

An inverse problem essentially involves finding parameters to a model so that it approximates an observed target phenomenon well. The latter is usually represented as a function in a metric space, while the approximating functions in our case are fixed points of a parametrized operator. Indeed, an iterated function system (IFS) and its associated weights induce a transfer operator over potential functions. For the direct problem to be well-posed, a unique fixed point of this operator must be shown to exist (while the inverse problem is not necessarily well-posed). This is therefore our primary focus for fractal potentials, which are the attractors of this transfer operator. An auxiliary bijective isometry is defined between the spaces of potential functions and probability measures, whereby the invariant / fractal potential corresponds to the well-known invariant measure of IFS.