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Newton polyhedron and hypersurfaces in toric varieties

With a compact smooth toric variety M and a fixed positive 0-cycle A (a fixed finite set of points equipped with positive integral multiplicities) belonging to the union M^1 of one-dimensional orbits of M one can associate the following problem: *find all hypersurfaces $H \subset M$ such that H does not pass through null-dimensional orbits and the intersection of H with M^1 is the 0-cycle A.*

This problem was solved in the case when $\dim M = 2$ in [1]. Let M_O be the closure in M of an orbit O. Let A_O be the 0-cycle $A \cap M_O$.

Theorem. *The problem has at least one solution H if and only if for each two-dimensional orbit O the problem for the toric surface M_O and the 0-cycles A_O has at least one solution.*

Moreover the intersection of any solution H with the torus $(\mathbb{C}^*)^n$ can be defined by equation $Q = 0$ where Q is a Laurent polynomial whose Newton polyhedron Δ and coefficients at monomials belonging to edges of Δ can be found explicitly and whose coefficients at all other monomials in Δ are arbitrary complex numbers.

References

1. A. Khovanskii. Newton polygons, curves on torus surfaces, and the converse Weil theorem, Russian Math. Surveys 52 (1997), no. 6, 1251-1279.