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The space of graded Noether normalizations of a graded algebra

Let $S = k[x_1, \ldots, x_n]$, k a field, and let J be a homogeneous ideal of S, with $d = \dim S/J$ and $c := n - d =$ height J. It is known that a sequence of linear forms $y_1, \ldots, y_d \in \mathrm{Span}_k\{x_1, \ldots, x_n\}$ induces a Noether normalization $k[y_1, \ldots, y_d] \to S/J$ of S/J if and only if any basis of the k-vector space H they generate do so, and that this in turn is determined by the c-dimensional dual space $H^\vee = \{v \in k^n : f(v) = 0 \quad \forall f \in H\}$. Thus viewing graded Noether normalizations as a subset of the Grassmannian $\mathbb{G}(c, n)$, let V be the complement of this set in the Grassmannian. Then V is a closed subvariety of $\mathbb{G}(c, n)$. Its defining ideal in the homogeneous coordinate ring D of $\mathbb{G}(c, n)$ (itself a well-behaved, though generally not regular, ring) has height 1, but we show that it otherwise reflects many of the properties of minimal primary decompositions of J itself. Indeed, we determine a well-behaved map that carries ideals of S to (not necessarily radical) ideals of D, which, especially when $k = \mathbb{C}$, is particularly well-behaved on homogeneous ideals of height $\geq c$ that are saturated with respect to S_+, and whose vanishing set is precisely the non-Noether normalization variety above. In particular, when $c = 1$, then $D \cong S$, and our map carries such an ideal J to an isomorphic copy of itself. This work is joint with Rebecca Goldin.