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Simplicial Homology Splitting

If $S = k[x_1, \ldots, x_n]$ is a polynomial ring over a field k and I is a graded ideal of S with minimal free resolution

$$0 \to \oplus_{j\in\mathbb{N}} S(-j)^{\beta_{p,j}} \to \oplus_{j\in\mathbb{N}} S(-j)^{\beta_{p-1,j}} \to \cdots \to \oplus_{j\in\mathbb{N}} S(-j)^{\beta_{1,j}} \to S,$$

then for each i and j, the rank $\beta_{i,j}(S/I)$ of the free S-modules appearing above are called the **graded Betti numbers** of the S-module S/I.

It is known that, in the class of monomial ideals, betti numbers can be interpreted as the homology of objects in discrete topology like simplicial complexes, order complexes of lattices, and more. In this talk, we will consider the question of when (or if) can we **split** up non-vanishing homology in these objects. In particular, if a simplicial complex Δ has non-vanishing homology in some dimension, then can the corresponding homological cycle induce homological cycles of smaller dimension? We will also see how splitting simplicial homology idea is related to the **sub-additivity** property of syzygies and use it to prove the sub-additivity property for some classes of monomial ideals.