AARMS-CMS Student Poster Session
Présentations par affiches des étudiants - AARMS-SMC
(Org: **Adèle Bourgeois** (University of Ottawa) and/et **Kaveh Mousavand** (Université du Québec à Montréal))

KYLE BRYENTON, University of Prince Edward Island
Exactly Solvable Anharmonic Potentials with Variable Bumps and Depths

A new approach based on Darboux Transformations is introduced to generate new classes of solvable anharmonic potentials with a variable number of bumps and depths. By introducing the concept of a transformation key, we present a method of controlling the number of bumps and their depths in these potentials. Although this method was applied to the one-dimensional generalized harmonic oscillator potential, it can be easily adapted to generate exactly-solvable potentials using other known quantum potentials.

POSPISIL CHRISTINA, University of Massachusetts Boston
Generalization Theory for Linear Algebra I

An algorithm for multiplying and adding matrices regardless of dimensions via an embedding is presented. An equivalent embedding for a general determinant theory is also investigated (Part I: Appropriate Inverses for non-injective mappings in one dimension are presented). In future work there will be applications to physics and other natural sciences be explored.

BERNAT ESPIGULÉ, Universitat de Barcelona
Complex Trees: Structural Stability of Connected Self-similar Sets

The theory of complex trees is introduced as a new approach to study a broad class of self-similar sets which includes Cantor sets, Koch curves, Lévy C curves, Sierpinski gaskets, Rauzy fractals, plane-filling curves, and fractal dendrites. We note a fundamental dichotomy for n-ary complex trees that allows us to study topological changes in regions where one-parameter families of connected self-similar sets are defined. Moreover, we show how to obtain these families from systems of equations encoded by tip-to-tip equivalence relations. The parameter space maps that we introduce to study these families of connected self-similar sets are new. For $T_A(z) := T\{z, \frac{1}{2}, \frac{1}{4z}\}$ we show that the boundary surrounding structurally stable trees is piecewise smooth.

KOLJA KYPKE, University of Guelph
Topological Climate Change

Climate change is undoubtedly one of the most impactful crises affecting humanity today. An increase in carbon emissions due to human activity induces a slew of additional climate effects such as rising sea levels, increase in extreme weather events, and more. The Earth's climate is a complicated combination of interdependent parts and behaves in a highly nonlinear fashion. Applying the mathematical theory of bifurcation to climate science allows for a new lens through which to view the challenges posed when attempting to model the climate. Such a model is better tuned to forecast major changes in the climate, as opposed to methods that capture gradual variations. An energy balance model in the form of a two-dimensional dynamical system is reduced to a single dimension on an centre manifold and transformed into the normal form of a cusp bifurcation. Four parameters, the carbon dioxide concentration of the atmosphere; the meridional ocean heat transport; the relative humidity; and an albedo switch function smoothness parameter; are investigated as potential unfolding parameters of the codimension-2 cusp bifurcation. The analysis shows that the most suitable choice for the application to paleoclimates is the carbon dioxide concentration and meridional ocean heat transport parameter combination. The paleoclimate problems of the Warm Equable Climate, Eocene-Oligocene Transition and the Pliocene Paradox are resolved by nature of the bistability and hysteresis exhibited by the cusp bifurcation. The appearance of significantly different climates states for the paleoclimates are due to the topological inequivalence of these states.

ANDREW LAVIGNE, Concordia
Abstract BMO Measure Spaces

Since BMO spaces where introduced by John and Nirenberg, they have been widely developed over spaces other than the Euclidean space, for example Manifolds or spaces of homogeneous type. The goal of this work is to study a very general BMO type space, with conditions imposed only on the measure and on the cover replacing balls. In this setting, assuming only a positive, strictly localizable measure, finite and non-zero on the chosen cover sets, necessary and sufficient conditions where found for this BMO space to be a Banach Space. These conditions are related to the cover sets only and don't require further restriction on the measure itself. Furthermore, other properties of the original BMO space where found to still hold in this setting.

KENZY ABDEL MALEK, Concordia University
Computable Metric Spaces

We review analysis analogues to notions from computability theory found in the literature, namely the definition of a computable metric space. An example of such a space is $C[0,1]$ with the sup metric and an appropriate dense, computable set of functions. Moreover, since the real Hardy space $H^p(\mathbb{R})$ is also a metric space for $0 < p \leq 1$, the goal is to ultimately define a computability structure on $H^p(\mathbb{R})$ using atomic decomposition

DAVID MIYAMOTO, University of Toronto
Basic forms on foliated manifolds

Given a foliated manifold $(M, \mathcal{F})$, a differential form α on M is called *basic* if $\iota_v \alpha = 0$ and $\iota_v d\alpha = 0$ for all tangent vectors v along the foliation. This gives the de Rham complex of basic forms $\Omega^\bullet_{\mathcal{F}}(M)$. Equipping the leaf space $M/\mathcal{F}$ with the quotient diffeology, we may also consider the de Rham complex $\Omega^\bullet(M/\mathcal{F})$ of diffeological differential forms. Using the fact the pseudogroup of diffeomorphisms associated to the (unique up to Morita equivalence) étale holonomy groupoid is countably generated, we prove that the quotient map $\pi : M \to M/\mathcal{F}$ induces an isomorphism $\pi^* : \Omega^\bullet(M/\mathcal{F}) \to \Omega^\bullet_{\mathcal{F}}(M)$. First we pass from the notion of basic forms with respect to a foliation, to basic forms on the object manifold of a Lie groupoid. We then use the fact this notion of basic is invariant under Morita equivalence of Lie groupoids to pass to the étale holonomy groupoid and its associated pseudogroup.

HERMIE MONTERDE, University of Manitoba
On the sum of strictly k-zero matrices

Let k be an integer such that $k \geq 2$. An n-by-n matrix A is said to be strictly k-zero if $A^k = 0$ and $A^m \neq 0$ for all positive integers m with $m < k$. Suppose A is an n-by-n matrix over a field with at least three elements. We show that if A is a nonscalar matrix with zero trace, then i) A is a sum of four strictly k-zero matrices for all $k \in \{2, \ldots, n\}$; and ii) A is a sum of three strictly k-zero matrices for some $k \in \{2, \ldots, n\}$. We prove that if A is a scalar matrix with zero trace, then A is a sum of five strictly k-zero matrices for all $k \in \{2, \ldots, n\}$. We also determine the least positive integer m such that every square complex matrix A with zero trace is a sum of m strictly k-zero matrices for all $k \in \{2, \ldots, n\}$.

GINA FARAJ RABBAH, York University
The Impact of Equilibrium Points on the Shape of Hysteresis Loops

Hysteresis is a phenomenon found in many natural dynamical systems which is typically described as a looping behaviour in the system's input-output graph. For a dynamical system to exhibit hysteresis, it must have multiple stable equilibria. This project examines the impact that different types of stability can have on the shape of the hysteretic loop exhibited in input-output graphs of Ordinary Differential Equations.

TEDI RAMAJ, Western University
A Dynamical Systems Approach to Modelling Competition Between Invasive Weeds and Native Plants

We explore the impact of invasive weeds on native forests by using partial differential equation (PDE) modelling and dynamical systems techniques. We perform this study via competition models where we consider both the weed and native species as functions of position and time. We are primarily interested in the existence of traveling wave solutions to the PDE systems. In particular, we study the existence of a so-called extinction wave which indicates the transition of the forest from a weed-free steady state to a steady state in which the native plant biomass is depleted. We also consider the existence of a traveling wave indicating a transition to a co-existence steady state. We put conditions on the parameters of our model which will be sufficient for the existence of the traveling wave fronts and we further find bounds on the wave speeds. We perform some simulations to support our analytic results. In all cases, we give a discussion on the ecological interpretations of our results, as well as some ideas on how to implement controls on invasive weed growth.

QUANLI SHEN, University of Lethbridge
The fourth moment of quadratic Dirichlet L-functions

We study the fourth moment of quadratic Dirichlet L-functions at $s = \frac{1}{2}$. We show an asymptotic formula under the generalized Riemann hypothesis, and obtain a precise lower bound unconditionally.

AARON SLOBODIN, University of Victoria
An Automated Employee Timetabling System

Employee scheduling is a difficult challenge for many business owners. In this poster presentation, we outline a user-friendly automated timetabler based on a three-dimensional integer linear program that accommodates employee seniority, shift type, labour laws, and vacation requests while optimizing for employee availability preferences. This new system builds upon our previous work by taking into account hourly floor constraints for status employees, predesignated shifts, and a bonus system to assign employees consecutive days off. This method has successfully been implemented at the retailer, Mountain Equipment Co-op, in Victoria, BC; a store with over 100 employees, 300 shift types, 40 possible roles, and 4 possible employee status tiers.

AFRODITI TALIDOU, University of Toronto
The FitzHugh-Nagumo equations on the surface of a warped cylinder. Stability of pulses

We study the FitzHugh-Nagumo system of equations on a long, thin cylinder that represents the outer membrane of the axon. This results in a system of partial differential equations in two spatial variables (plus time). Key questions are the existence of a pulse - a special solution that travels along the length of the axon - and its stability under small perturbations of the initial conditions and the geometry. These questions have been studied extensively for the simpler model where the axon of the neuron has no width.

We use classical and modern techniques from the theory of partial differential equations to address these issues in the more realistic geometric framework where the axon has the shape of a long, thin cylinder that varies along its length. To support and complement our theoretical results, we are also simulating the system numerically.