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The universal invariant profile of the multiplicative group

The structure of the multiplicative group $M_n = (\mathbb{Z}/n\mathbb{Z})^\times$ encodes a great deal of arithmetic information about the integer n (examples include $\phi(n)$, the Carmichael function $\lambda(n)$, and the number $\omega(n)$ of distinct prime factors of n). We examine the invariant factor structure of M_n for typical integers n, that is, the decomposition $M_n \cong \mathbb{Z}/d_1\mathbb{Z} \times \mathbb{Z}/d_2\mathbb{Z} \times \cdots \times \mathbb{Z}/d_k\mathbb{Z}$ where $d_1 \mid d_2 \mid \cdots \mid d_k$. We show that almost all integers have asymptotically the same invariant factors for all but the largest factors; for example, asymptotically $1/2$ of the invariant factors equal $\mathbb{Z}/2\mathbb{Z}$, asymptotically $1/4$ of them equal $\mathbb{Z}/12\mathbb{Z}$, asymptotically $1/12$ of them equal $\mathbb{Z}/120\mathbb{Z}$, and so on. Furthermore, for positive integers k, we establish a theorem of Erdős–Kac type for the number of invariant factors of M_n that equal $\mathbb{Z}/k\mathbb{Z}$, except that the distribution is not a normal distribution but rather a skew-normal or related distribution. This is joint work with Reginald M. Simpson.