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On the Duffin-Schaeffer conjecture

Let $\mathcal{S}$ be a set of natural numbers. We wish to understand how well we can approximate a "typical" real number using reduced fractions whose denominator lies in $\mathcal{S}$. To this end, we associate to each $q \in \mathcal{S}$ an acceptable error $\Delta_q > 0$. When is it true that almost all real numbers (in the Lebesgue sense) admit an infinite number of reduced rational approximations a/q, $q \in \mathcal{S}$, within distance Δ_q? In 1941, Duffin and Schaeffer proposed a simple criterion to decide whether this is case: they conjectured that the answer to the above question is affirmative precisely when the series $\sum_{q\in\mathcal{S}} \phi(q)\Delta_q$ diverges, where $\phi(q)$ denotes Euler's totient function. Otherwise, the set of "approximable" real numbers has null measure. In this talk, I will present recent joint work with James Maynard that settles the conjecture of Duffin and Schaeffer.