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No set of spaces detects isomorphisms in the homotopy category

Whitehead's theorem says that a map of pointed, connected CW complexes is a homotopy equivalence if and only if it induces an isomorphism on homotopy groups.

In the unpointed setting, one can ask whether there is a set $\mathcal{S}$ of spaces such that a map $f : X \to Y$ between connected CW complexes is a homotopy equivalence if and only if it induces bijections $[A, X] \to [A, Y]$ for all A in $\mathcal{S}$. Heller claimed that there is no such set $\mathcal{S}$, but his argument relied on an "obvious" statement about weak colimits in the homotopy category of spaces. We show that this obvious statement is false, thus reopening the question above. We then show that Heller was in fact correct that no such set $\mathcal{S}$ exists, using a different, more direct method.