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On rank-one unitary perturbations of the shift operator over a de Branges-Rovnyak space

Due to a celebrating Theorem of Beurling and Lax, the closed subspaces K_θ, called the model spaces and parametrized by some function θ, of the Hardy space H^2 characterize the closed invariant subspaces of the backward shift operator S^* on the Hardy space H^2. This means that any closed subspace V of H^2 such that $S^*V \subset V$ must be a model space K_θ, that is $V = K_\theta$.

In connection with this result, in 1972, Clark studied the invariant subspaces of the restricted shift $S_\theta := P_{K_\theta} \, S|_{K_\theta}$ defined on the model space K_θ where P_{K_θ} is the orthogonal projection on K_θ. One of his interests was to characterize the rank-one unitary perturbations of the restricted shift operator S_θ. He showed that such operators must be of the form U_α where $U_\alpha f = S_\theta f + \langle \alpha f, S^*\theta \rangle \, 1$ where $\langle \cdot, \cdot \rangle$ denotes the inner product in H^2 and $|\alpha| = 1$.

The goal of the talk is to generalize Clark's result to the de Branges-Rovnyak spaces in the case of extreme points b, a generalisation of the model space K_θ. More specifically, I will introduce the de Branges-Rovnyak spaces and show how the shift operator may be defined on this space. Then, I will show that Clark's result still hold in this framework.